Pythagorean Theorem Investigation: It’s As Easy As… a, b, c

**Overview of the Activity:**

Many times the Pythagorean Theorem is presented as simply a procedural way to find the length of the missing side of a right triangle. There is much more to it than just that. We look at an intuitive way of understanding the theorem, analyze proofs, and apply it to real world scenarios in this document of good classroom tasks and problems.

**Common Core Standard(s) Addressed:**

[CCSS.MATH.CONTENT.8.G.B.6](http://www.corestandards.org/Math/Content/8/G/B/6/)  
Explain a proof of the Pythagorean Theorem and its converse.

[CCSS.MATH.CONTENT.8.G.B.7](http://www.corestandards.org/Math/Content/8/G/B/7/)  
Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

[CCSS.MATH.CONTENT.8.G.B.8](http://www.corestandards.org/Math/Content/8/G/B/8/)  
Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

**Prerequisites:**

Students should know how to solve quadratics by using the inverse operation of square roots.

**Time Required:**

The time required depends on how many of the tasks and problems you choose to use. If all of the problems are completed in class it would likely take two 50 minute class periods.

**Materials Required:**

* Scissors
* Colored markers/crayons/pencils
* Calculators
* Some problems require internet access

**Lesson Details:**

This “lesson” is really a series of good classroom tasks and problems designed to help students understand and apply the Pythagorean Theorem. As many or as few of the problems as is desired may be used. Each of the standards listed above are addressed in at least one of the problems.

Instructor Guide

Pythagorean Theorem Investigation: It’s As Easy As… a, b, c

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Near the center of the grid below form a *right angle* by drawing one 3-unit length and one 4-unit length. After drawing the two lengths, create a *right triangle* by connecting the ends of the two lengths with a third side, called side “c”. This side is called the ***hypotenuse***. Label the 3-unit length as side “a” and the 4-unit length as side “b”. These are called the ***legs***.

*See below*

1. With your partner and **WITHOUT** measuring, make a conjecture as to which is longer: a + b or c or if they would be the same size and explain your reasoning.

Conjecture (circle one): ***a + b is longer*** c is longer a + b and c are the same length

Rationale:

*It appears that if you add the two smaller sides lengths together that they will be longer than the hypotentuse.*

1. Choose one color. Using side “a” form a square on the diagram you have already created below. Choose another color and using side “b” form a second square on the same diagram.

*See below.*

1. Cut out only the colored squares you just created. Using only these squares (you may need to cut them up into smaller squares), create one bigger square. What do you notice about this largest square and the other two original squares? How does the side of the hypotenuse relate to the side of the new triangle you created? *Students here are to “see” that the sum of the squares of the two smaller squares equals the square of the hypotenuse*

*c*

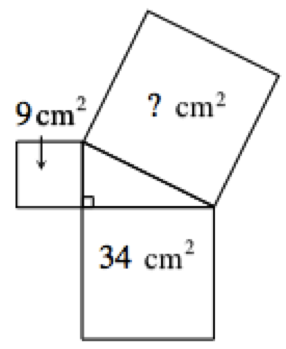
*a = 3*

*b = 4*

**Pythagorean Theorem**: states that in a right triangle, the length of one leg squared plus the length of the other leg squared is equal to the length of the hypotenuse squared.  It can be written as an equation like this:

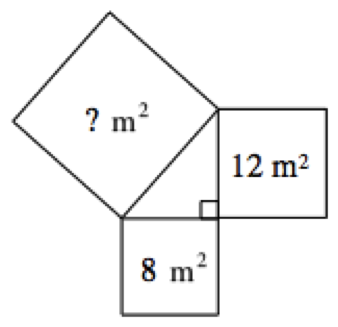
(leg a)2 + (leg b)2 = (hypotenuse)2

1. Use the Pythagorean Theorem to find the missing value for each diagram below.



5 ft

? ft



36 cm2

64 cm2

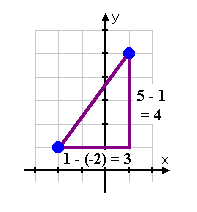
?? cm2

13 ft

1. Using the diagram provided, demonstrate why the Pythagorean Theorem ( ) and the distance formula ( are equivalent.





**(1,5)**

**(-2,1)**

1. Find the distance between the points located on the coordinate plane at (6, 8) and (-4, -2). Show on a figure how the Pythagorean Theorem was used.

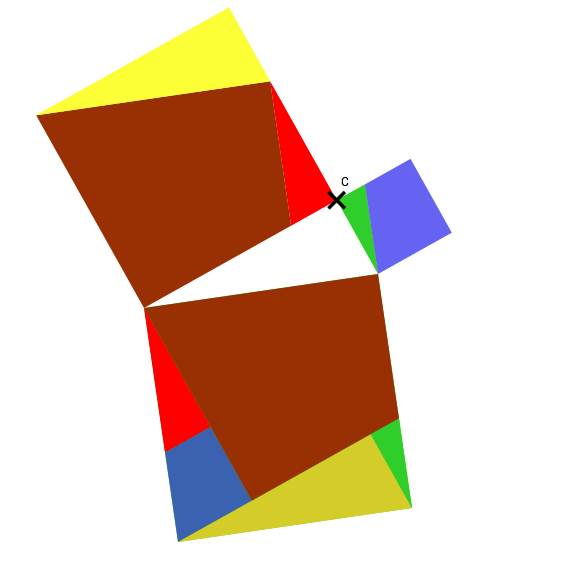
**(6, 8)**

**.**

**.**

**(-4, -2)**

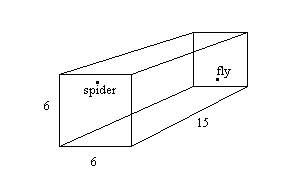


1. Go to <http://www.cut-the-knot.org/pythagoras/> and demonstrate proof 3, 4, 5, or 10 by using cutout triangles you create. Include the algebraic support alongside the manipulatives and convince a classmate that the Pythagorean theorem is true.
2. The following figure is from [TedCoe.com](http://tedcoe.com/math/geometry/wordless-pythagorean). Explain why the “Wordless Pythagorean” should be convincing evidence that the Pythagorean Theorem is true.

*Assuming that the colors are coordinated areas, then if you add the area of the square comprised of the yellow, brown, and red regions to the area of the square comprised of the blue and green regions, you will notice that the largest square is comprised of all of those regions.*

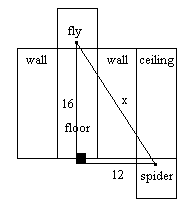
1. Watch the clip from the Wizard of Oz at <https://www.youtube.com/watch?v=jbvip1Ot6jQ> showing the Scarecrow getting his brain from the Wizard. Explain what the error is in his quoting the Pythagorean Theorem and what he SHOULD have said instead. Use a diagram (or diagrams) to compare and contrast the incorrect quote and the actual theorem.

*The Scarecrow points to his temple and says “the sum of the square roots of two sides of an isosceles triangle is equal to the square root of the remaining side.” The are multiple errors. First, it is not the sum of the* ***square roots*** *of the two sides but the sum of the* ***squares*** *of the two sides that is equal to the* ***square*** *of the remaining side. Second, the triangle does not have to be isosceles (meaning having two sides the same length) but simply a right triangle. Finally, the sides we choose to square and sum matter. We must select the legs to square and sum and then equate that to the square of the hypotenuse. Therefore, what he should have quoted was, “the sum of the squares of the two legs of a right triangle is equal to the square of the hypotenuse.”*

1. Challenge/Extension problem: After viewing (and reviewing): [a video of the Pythagorean Theorem in 3D](https://learnzillion.com/lessons/1303-apply-the-pythagorean-theorem-to-three-dimensional-figures-using-right-triangles), solve the following problem. Note the video is a hint for what may need to be done to solve the problem.

A spider and a fly are in the same room. The fly is fast asleep and the spider is hungry. What is the shortest route the spider can take to get the fly? One stipulation is that the spider must crawl along the walls, ceiling, and/or floor and may not slingshot herself across the room. The room is 15 meters long, 6 meters wide, and 6 meters high as shown. The spider is 0.5 meters from the ceiling and 3 meters from each wall. The fly is 0.5 meters from the floor and 3 meters from each wall. Another hint: The answer is NOT 21 meters.

*The problem is complicated since it is in three dimensions. Sooner or later the student may try the equivalent problem in two dimensions. The room is flattened out and one has to pick the best net. The shortest way between two points is the straight line between them. Now, how can we find this distance, when all we know is the vertical and horizontal displacement? Now, isn't that an interesting question?*

**

*That the answer is 20 m using the Pythagorean Theorem and it transcends the common sense of many students.*

Student Activity

Pythagorean Theorem Investigation: It’s As Easy As… a, b, c

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Near the center of the grid below form a *right angle* by drawing one 3-unit length and one 4-unit length. After drawing the two lengths, create a *right triangle* by connecting the ends of the two lengths with a third side, called side “c”. This side is called the ***hypotenuse***. Label the 3-unit length as side “a” and the 4-unit length as side “b”. These are called the ***legs***.
2. With your partner and **WITHOUT** measuring, make a conjecture as to which is longer: a + b or c or if they would be the same size and explain your reasoning.

Conjecture (circle one): a + b is longer c is longer a + b and c are the same length

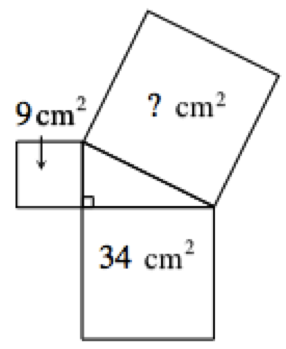
Rationale:

1. Choose one color. Using side “a” form a square on the diagram you have already created below. Choose another color and using side “b” form a second square on the same diagram.
2. Cut out only the colored squares you just created. Using only these squares (you may need to cut them up into smaller squares), create one bigger square. What do you notice about this largest square and the other two original squares? How does the side of the hypotenuse relate to the side of the new triangle you created?

**Pythagorean Theorem**: states that in a right triangle, the length of one leg squared plus the length of the other leg squared is equal to the length of the hypotenuse squared.  It can be written as an equation like this:

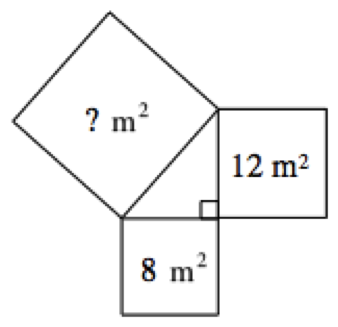
(leg a)2 + (leg b)2 = (hypotenuse)2

1. Use the Pythagorean Theorem to find the missing value for each diagram below.



5 ft

? ft



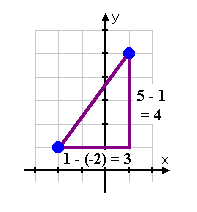
36 cm2

64 cm2

?? cm2

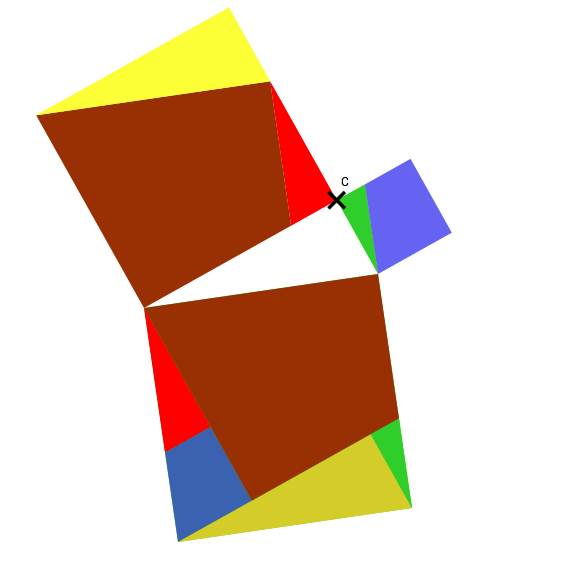
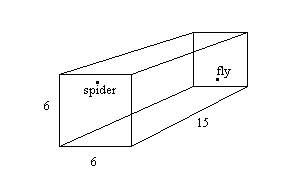
13 ft

1. Using the diagram provided, demonstrate why the Pythagorean Theorem ( ) and the distance formula ( are equivalent.



**(1,5)**

**(-2,1)**

1. Find the distance between the points located on the coordinate plane at (6, 8) and (-10, -2). Show on a figure how the Pythagorean Theorem was used.
2. Go to <http://www.cut-the-knot.org/pythagoras/> and demonstrate proof 3, 4, 5, or 10 by using cutout triangles you create. Include the algebraic support alongside the manipulatives and convince a classmate that the Pythagorean theorem is true.
3. The following figure is from [TedCoe.com](http://tedcoe.com/math/geometry/wordless-pythagorean). Explain why the “Wordless Pythagorean” should be convincing evidence that the Pythagorean Theorem is true.
4. Watch the clip from the Wizard of Oz at <https://www.youtube.com/watch?v=jbvip1Ot6jQ> showing the Scarecrow getting his brain from the Wizard. Explain what the error is in his quoting the Pythagorean Theorem and what he SHOULD have said instead. Use a diagram (or diagrams) to compare and contrast the incorrect quote and the actual theorem.
5. Challenge/Extension problem: After viewing (and reviewing): [a video of the Pythagorean Theorem in 3D](https://learnzillion.com/lessons/1303-apply-the-pythagorean-theorem-to-three-dimensional-figures-using-right-triangles), solve the following problem. Note the video is a hint for what may need to be done to solve the problem.

A spider and a fly are in the same room. The fly is fast asleep and the spider is hungry. What is the shortest route the spider can take to get the fly? One stipulation is that the spider must crawl along the walls, ceiling, and/or floor and may not slingshot herself across the room. The room is 15 meters long, 6 meters wide, and 6 meters high as shown. The spider is 0.5 meters from the ceiling and 3 meters from each wall. The fly is 0.5 meters from the floor and 3 meters from each wall. Another hint: The answer is NOT 21 meters.