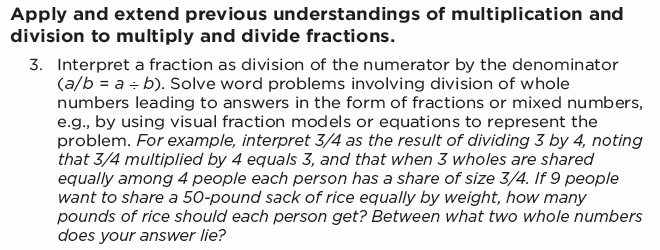
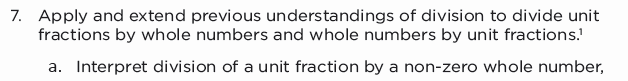
**Dividing Fractions**

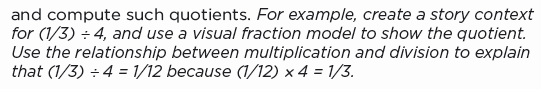
This set of activity activities are designed to help students to structure their thinking so that they are afforded the opportunity to make sense of the idea of division of fractions. The goal of this collection of activities is to allow the division algorithm to emerge from quality thinking and reasoning about the mathematics needed to divide fractions. While students will engage in many (all?) of the Standards for Mathematical Practices, the primary practice that is the focus of these activities is Practice #8: *Look for and express regularity in repeated reasoning*. As a result of developing this and the other mathematical practices, division of fractions will become part of a well-connected network of understanding.

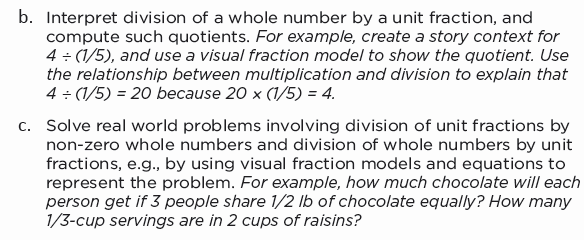
The activities in this series of lessons have just a few questions in each. However, each question will likely take much time and were designed to elicit much conversation, discussion, debate, explanation, etc. That is, students should be thinking deeply about what they are doing and why they are doing it. Teachers should be challenging students to explain what “it” means as students describe the methods they used to answer each question.

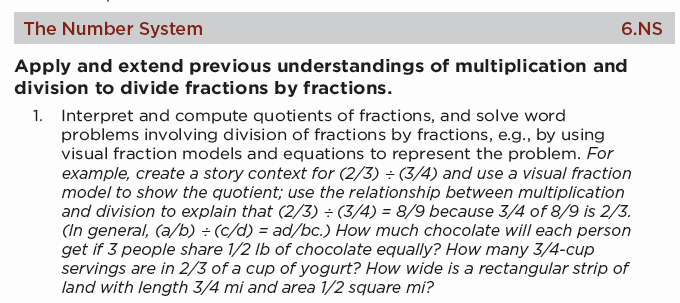
This is also an excellent example of why the worksheets alone may not accomplish the intended goal if the user does not understand the purpose. By combining the workshop experience with the solutions, my hope is that the worksheets will accomplish the intended goal: for students to engage in repeated reasoning for the purpose of creating an algorithm for dividing fractions.











**Part 1**

1. I love Cowboy Cookies. The recipe calls for 1 cup of flour. At my house, measuring cups are hard to find and on the day I wanted to bake Cowboy Cookies, I could only find a cup measuring cup (see picture of my kitchen drawer). Describe how I can, as accurately as possible, measure 1 cup of flour using only the cup measuring cup. Show and/or describe all reasoning needed to resolve the situation. What mathematical operation is at play in this situation?

The question is: how many copies of cup are needed to make 1 cup? Computationally speaking, we need to compute .

The goal is to think about this computation and not resort to a procedure or algorithm.

The thinking might sound something like this: We need 3 copies of, or 3- or to make a full cup of flour.

So the answer to the questions is 3; we need to fill our cup measuring cup three times in order to add the required amount of flour.

2. Now suppose that the only measuring cup I could find is the cup measuring cup. Remember that the recipe calls for 1 cup of flour. Describe how I can, as accurately as possible, measure 1 cup of flour using only the cup measuring cup. Show and/or describe all reasoning needed to resolve the situation. What mathematical operation is at play in this situation?

The question is: how many copies of cup are needed to make 1 cup? Computationally speaking, we need to compute .

The goal is to think about this computation and not resort to a procedure or algorithm.

Compared to #1, the measuring cup is twice as large, so we should use half as much in order to meet the requirement of adding 1 cup of flour. In #1, we needed 3 copies of of a cup. Now, we need half as many copies of of a cup. Half of 3 is . We need filled cup of flour to meet the requirement.

Another way of thinking: we might see as 2 copies of . So, if we fill the cup half full, it will contain cup of flour. We need 1 filled cup plus an additional half-way filled cup or filled cup of flour to meet the requirement.

3. Describe the relationship between the following two operations as they relate to the cookie situations presented in #1 and #2.

 and 

Push students to articulate that  is two times as large as . Also, push students to say thatis half as large as . We can connect this thinking to the situation. Since the cup is twice as large as the cup, we need to use half as many filledcup of flour to meet the goal compared to thecup. Also, sincecup is half as large ascup, we will need twice as many filled cups of flour to meet the requirement.

That is:

 and .

4. Now suppose that I could only find a cup measuring cup. Describe how I can, as accurately as possible, measure 1 cup of flour using only the cup measuring cup. Show and/or describe all reasoning needed to resolve the situation. What mathematical operation is at play in this situation?

The question is: how many copies of cup are needed to make 1 cup? Computationally speaking, we need to compute . The goal is to think about this computation and not resort to a procedure or algorithm.

The thinking might sound something like this: We need 4 copies of or 4- or to make a full cup of flour.

So the answer to the questions is 4; we need to fill our cup measuring cup four times in order to add the required amount of flour.

5. Now suppose that the only measuring cup I could find is the cup measuring cup. Remember that the recipe calls for 1 cup of flour. Describe how I can, as accurately as possible, measure 1 cup of flour using only the cup measuring cup. Show and/or describe all reasoning needed to resolve the situation. What mathematical operation is at play in this situation?

The question is: how many copies of cup are needed to make 1 cup? Computationally speaking, we need to compute . The goal is to think about this computation and not resort to a procedure or algorithm.

Compared to #4, the measuring cup is three times as large, so we should use one-third as much in order to meet the requirement of adding 1 cup of flour. In #4, we needed 4 copies of of a cup. Now, we need one-third as many copies of of a cup. One-third of a copy of 4 is. We need  filled cup of flour to meet the requirement.

Another way of thinking: we might see as 3 copies of . So, if we fill the cup one-third full, it will containcup of flour. We need 1 filled cup plus an additional one-third of the way filled cup or  filledcup of flour to meet the requirement.

6. Describe the relationship between the following two operations as they relate to the cookie situations presented in #1 and #2.

 and 

Push students to articulate that  is three times as large as . Also, push students to say that is one-third as large as . We can connect this thinking to the situation. Since a cup is three times as large as a cup, we need to use one-third as many filled cup of flour to meet the goal compared to the cup. Also, since cup is one-third as large as cup, we will need three times as many filled cups of flour to meet the requirement.

That is:

 and .

7. In general, suppose I have a measuring cup that can hold (hypothetically) cups of flour and that the recipe still calls for 1 cup of flour. How many filled cups of flour will be needed to meet the recipe requirements? Express this situation in mathematical symbols.

We need *b* copies of filled cups of flour to meet the recipe requirements. In mathematical symbols, the situation is saying that .

8. Now suppose I have a measuring cup that contains (hypothetically) cups of flour and that the recipe still calls for 1 cup of flour. How many filled cups of flour will be needed to meet the recipe requirements? Express this situation in mathematical symbols. Hint: use your reasoning in #1-3 to guide you.

Since is *a* times as large as , we will need times as many filled cups of flour to meet the requirement. That is,



Note that . That is, dividing by the fraction is equivalent to multiplying by the fraction .

**Part 2**

Use the ***reasoning*** developed so far to think through and to respond to each of the following situations. That is, refrain from resorting to the use of a procedure or algorithm. Rather, think about the relationships between these new situations and the situations you have thought about previously in this series of tasks.

1. A recipe calls for 2 cups of flour. You only have a cup measuring cup. How many filled cup measuring cups are needed to meet the requirements of the recipe?

We know that when we needed 1 cup of flour, 3 filled cups were needed; . Since we need twice as much flour, (1 cup to 2 cups), we will need to fill the cup measuring cup twice as many times.



We need 6 filled cup measuring cups to meet the requirements of the recipe.

2. A recipe calls for 3 cups of flour. You only have a cup measuring cup. How many filled cup measuring cups are needed to meet the requirements of the recipe?

We know that when we needed 1 cup of flour, 3 filled cups were needed; . Since we need three times as much flour, (1 cup to 3 cups), we will need to fill the 1/3 cup measuring cup three times as many times.



We need 9 filledcup measuring cups to meet the requirements of the recipe.

3. A recipe calls for 2 cups of flour. You only have a cup measuring cup. How many filled cup measuring cups are needed to meet the requirements of the recipe?

We know that when we needed 1 cup of flour, 4 filled cups were needed; . Since we need twice as much flour, (1 cup to 2 cups), we will need to fill the cup measuring cup twice as many times.



We need 8 filled cup measuring cups to meet the requirements of the recipe.

4. A recipe calls for 3 cups of flour. You only have a cup measuring cup. How many filled cup measuring cups are needed to meet the requirements of the recipe?

We know that when we needed 1 cup of flour, 4 filled cups were needed; . Since we need three times as much flour, (1 cup to 3 cups), we will need to fill the 1/4 cup measuring cup three times as many times.



We need 12 filled cup measuring cups to meet the requirements of the recipe.

5. A recipe calls for *c* cups of flour. You only have a  cup measuring cup. How many filled cup measuring cups are needed to meet the requirements of the recipe?

We know that when we needed 1 cup of flour, *b* filled  cups were needed; . Since we need *c* times as much flour, (1 cup to *c* cups), we will need to fill thecup measuring cup *c* times as many times.



We need filledcup measuring cups to meet the requirements of the recipe.

6. A recipe calls for 2 cups of flour. You only have a cup measuring cup. How many filled cup measuring cups are needed to meet the requirements of the recipe?

We know that when we needed 1 cup of flour,  filledcups were needed; . Since we need twice as much flour, (1 cup to 2 cups), we will need to fill the cup measuring cup twice as many times.



We need 3 filled cup measuring cups to meet the requirements of the recipe.

7. A recipe calls for 3 cups of flour. You only have a cup measuring cup. How many filled cup measuring cups are needed to meet the requirements of the recipe?

We know that when we needed 1 cup of flour,  filled cups were needed; . Since we need three as much flour, (1 cup to 3 cups), we will need to fill the cup measuring cup three times as many times.



We need  filledcup measuring cups to meet the requirements of the recipe.

8. A recipe calls for 2 cups of flour. You only have a cup measuring cup. How many filled cup measuring cups are needed to meet the requirements of the recipe?

We know that when we needed 1 cup of flour,  filled cups were needed; . Since we need twice as much flour, (1 cup to 2 cups), we will need to fill thecup measuring cup twice as many times.



We need  filledcup measuring cups to meet the requirements of the recipe.

9. A recipe calls for 3 cups of flour. You only have a cup measuring cup. How many filledcup measuring cups are needed to meet the requirements of the recipe?

We know that when we needed 1 cup of flour,  filledcups were needed; . Since we need three times as much flour, (1 cup to 3 cups), we will need to fill thecup measuring cup three times as many times.



We need 4 filledcup measuring cups to meet the requirements of the recipe.

10. A recipe calls for *c* cups of flour. You only have an cup measuring cup. How many filledcup measuring cups are needed to meet the requirements of the recipe?

We know that when we needed 1 cup of flour,  filledcups were needed; . Since we need *c* times as much flour, (1 cup to *c* cups), we will need to fill thecup measuring cup *c* times as many times.



We need  filled cup measuring cups to meet the requirements of the recipe.

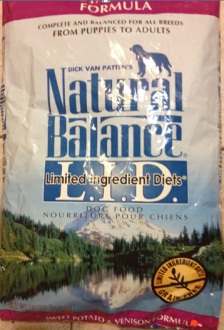
11. Write a concluding statement that describes the results of the reasoning and work you have completed in Part 2 of this activity.

Consider the statement from #10 - . This says that when computing , we can equivalently compute . That is, when we divide by a fraction, we can equivalently multiply by the reciprocal of that fraction.

**The following set of activities is designed to develop the idea of division of fractions from a proportional reasoning perspective.**

**1 – Focus on Proportional Reasoning**

Every morning when I wake up, I feed my two dogs, Tobie and Gracie. It seems like I am buying large bags of dog food so often that I wondered one morning…how many scoops of dog food do I deposit into their doggie dishes before running out of dog food?

I use a clear plastic cup (see picture) to scoop the dog food. I determined that it takes approximately 93 scoops (rounded to the nearest whole scoop) to exhaust the supply of dog food. I then need to go to the dog food store to buy a new 28 pound bag of food for Tobie and Gracie.

1. Draw a pair of line segments to represent the situation involving the number of scoops and the total amount of dog food in the bag. Label as much information on the segments as you can.

28 lbs

93 scoops

Imagine cutting the “scoops” line segment up into 93 equal parts. Also, imagine cutting the “pounds” line segment up into 93 equal parts.

93

93 scoops

93 scoops

2. If I cut the “scoops” line segment into \_\_\_\_\_\_\_\_\_\_\_\_\_ pieces, each piece represents

1

1/93

\_\_\_\_\_\_\_\_\_\_\_\_\_ copies of the total number of scoops or \_\_\_\_\_\_\_\_\_\_\_\_\_ scoops.

3. To keep the situation ***in proportion*** (you may want to discuss what this means), cut the

93

“pounds” line segment into \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_pieces where each piece represents

28/93

1/93

\_\_\_\_\_\_\_\_\_\_\_\_\_\_ copies of the total number of pounds of dog food or \_\_\_\_\_\_\_\_\_\_\_ pounds.

4. Represent, using fractions, the number of pounds of dog food that fit into one scoop. Explain how you know.

1. We are partitioning the 28 pounds into 93 equal parts. Therefore, we divide 28/93 to find how many pound of dog food in each scoop.

2. We need to find 1/93 copies of 28 pounds. That is, 1/93 \* 28 which is equivalent to 28/93.

3. Push the idea of scale factor.

**Activity 2 – The Dog Food Saga Continues**

In Activity 1, the number of scoops of dog food was rounded to the nearest whole number of scoops. In reality, it takes scoops before I run out of dog food and have to buy another 28 pound bag of dog food. As you respond to the items on this page, use fractions to represent all quantities.

1. Draw a pair of line segments to represent the situation involving the number of scoops and the total amount of dog food in the bag. Label as much information on the segments as you can.

92 3/4 scoops

28 lbs

Imagine cutting the “scoops” line segment up into equal parts. Also, imagine cutting the “pounds” line segment up into equal parts.



2. If I cut the “scoops” line segment into \_\_\_\_\_\_\_\_\_\_\_\_\_ pieces, each piece represents

1



\_\_\_\_\_\_\_\_\_\_\_\_\_ copies of the total number of scoops or \_\_\_\_\_\_\_\_\_\_\_\_\_ scoops.

3. To keep the situation ***in proportion*** (you may want to discuss what this means), cut the



“pounds” line segment into \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_pieces where each piece represents





\_\_\_\_\_\_\_\_\_\_\_\_\_\_ copies of the total number of pounds of dog food or \_\_\_\_\_\_\_\_\_\_\_ pounds.

4. Represent, using fractions, the number of pounds of dog food that fit into one scoop. Explain how you know. Simplify your result into a fraction of the form where *a* and *b* are whole numbers.

Answers will vary. Focus on meanings…focus on scale factor.

1.  Note that the idea of scaling and proportion are at the foundation of this response.

2. A student might observe that the denominator shows 371 copies of ¼ scoops (371(¼)). Using the idea of scale factor, they multiply numerator and denominator by 4 to scale up to full scoops - . That is, there are 112 pounds in 371 full scoops which is in proportion to 28 pounds in 371 quarter scoops. Then, students might express the number of pounds in just 1 scoop using the scale factor 1/371.



We can say that there are 112/371 pounds in one scoop of dog food. By the way…this is approximately 0.3 pounds per scoop.

**Activity 3 – Baking Cookies**

I love Cowboy Cookies. The recipe shown calls for cup of flour. At my house, measuring cups are hard to find and on the day I wanted to bake Cowboy Cookies, I could only find a cup measuring cup (see picture of my kitchen drawer). Describe how I can, as accurately as possible, measure cup of flour using only the cup measuring cup. Show and/or describe all reasoning needed to resolve the situation.



We need to determine how many 1/2 cups are in 2/3 cup. That is, how many copies of 1/2 are in 2/3? This is a division problem!



At this stage, students are encouraged to think about this using the idea of the scale factor.



There are 4/3 copies of 1/2 in 2/3. To make the cookies, 4/3 or 1 and 1/3 copies of 1/2 cup are required. That is, one would have to add 1 full ½ cup of flour plus an additional 1/3 of a full ½ cup.

Another strategy involving common denominators:



Students may think: how many 3/6 are in 4/6? With a multiplicative understanding of fraction, we have 3 copies of 1/6 and 4 copies of 1/6. So, we can think: how many copies of 3 are in 4?

**Activity 4 – Coffee Blends**

Some people like to create their own coffee blends by mixing together different kinds of freshly roasted and ground coffee beans. One website, www.thecaptainscoffee.com, recommends different blends for people to try. One such blend is shown.



Suppose a person has pounds of Ethiopian Harrar coffee that they wish to blend with Sumatran Mandeling (of which they have abundant supply). How many pounds of this blend can be made?

For every pound of the blended coffee, we will havelbs. of Sumatran Mandheling and lbs. of Ethiopian Harrar. The question becomes: how many lbs. are in pounds?



Note the idea of scaling. We can continue scaling:



We can make or pounds of coffee.