**Functions Module**

**An Activity Designed to Motivate Function Notation**

Adapted from Mathematics Teacher, August 2016, Reader Reflections, Albert Goetz and

Pat Thompson’s “Why use *f*(*x*) when all we really mean is *y*?”

The state of Virginia, in November 2012, introduced high-occupancy toll (HOT) lanes on parts of the Washington Beltway, the stretch of Interstate 495 that circumvents the nations’ capital, in an attempt to ease congestion on this road. Drivers who wish to pay a price (which varies directly with the amount of traffic) can opt to use special lanes with a higher speed limit and, at least for now, much less traffic. Is it time efficient to use these HOT lanes?

Suppose the length of the HOT lanes is 5 miles and that for the entire 5 miles drivers can average 65 MPH. For the regular lanes, let’s assume that for the same 5 miles traffic averages 35 MPH. To get to the HOT lanes, motorists must exit the regular lanes and take an overpass that puts them on the high-speed lanes, to which a traffic light controls access. Suppose the average wait time at this light is 2 minutes.

1. How much time is saved, in minutes, if a motorist takes the HOT lanes rather than the regular lanes? Be prepared to explain your results.

2. If the length of the HOT lanes were 1, 2.5, 3.7, or 4.2 miles in length, how much time, in minutes, is saved in each case? Be prepared to explain your results.

Note that the greater the length of the HOT lanes, the greater amount of time saved by using the HOT lanes. In the case of the 1 mile and 2.5 mile HOT lanes distance, the time saved is negative which means that it take more time to transition to the HOT lanes than to stay in the regular lanes.

3. If the length of the HOT lanes were 6.7, 7.1, or 8.5 miles in length, how much time, in minutes, is saved in each case? Be prepared to explain your results.

4. What formula are you using to calculate the time saved in each case above? Does knowing the formula help you to calculate values faster in each case above?

where *x* is the length, in miles, of the HOT lanes. No. Knowing the formula does not help us to calculate the values faster.

5. Listen to your instructor explain how to use the Desmos calculator to help with the calculations.

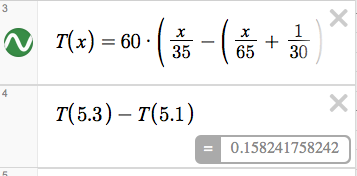
Tell the students that there is a way to name this formula and have the formula calculate itself automatically. Using desmos.com, define

Explain that *x* is the length of the HOT lanes in miles and *T*(*x*) produces the time saved, in minutes, by taking the HOT lanes rather than the regular lanes.

Type *T*(8.5). The display shows 4.72527. Have students interpret the display. Type *T*(2.5). The display shows -0.0219. Have students interpret the display.

Continue as long as you think is best.

6. How much does the time saved change were we to increase the length of the HOT lanes from 5.1 miles to 5.3 miles? Use the Desmos calculator to respond to this question. What will you enter into Desmos and why?



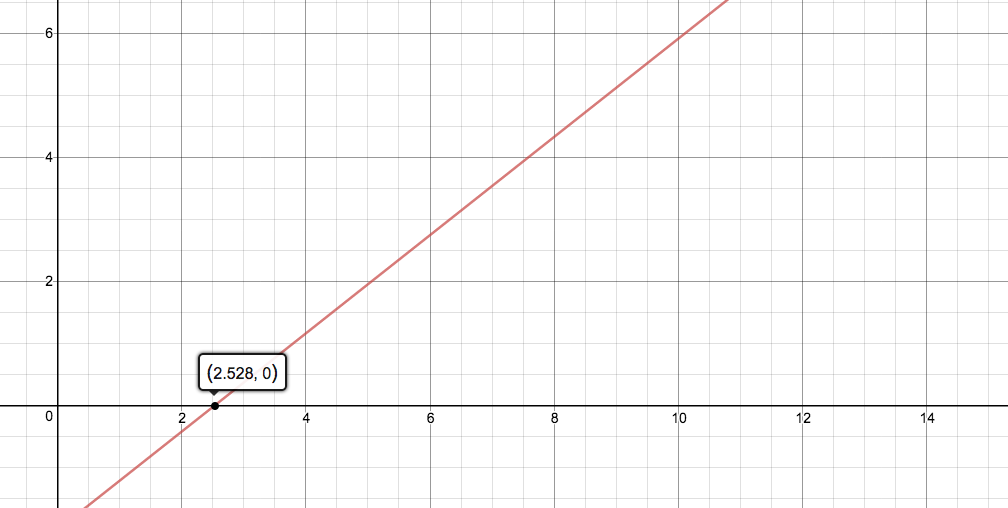
The amount of time saved increases by about 0.158 minutes if the length of the HOT lanes increases from 5.1 miles to 5.3 miles.

7. How might we think about what Desmos is doing when we type in *T*(number)?

It is important that students develop a mental model of what the device is doing to produce numbers. It won’t take long for them to propose something like a function machine.

8. How might we use our definition of *T* to graph the relationship between time saved and length of the HOT lanes? Create this graph. Then write a response to the initial problem statement: Is it time efficient to use these HOT lanes?

Imagine using desmos.com to compute many, many, many different scenarios and plotting each point. This collection of points is the graph of the function. It is time efficient to use the HOT lanes if the distance traveled on the HOT lanes is great enough. Students should be able to explain what is “great enough” using the graph. The graph shows that if the distance traveled in the HOT lanes (*x*) is greater than approximately 2.53 miles, then it is time efficient to use the HOT lanes. The desmos.com image is shown below.



9. Suppose the assumed speeds where 60 MPH using the HOT lanes and 40 MPH using the regular lanes. Must we start over from scratch in order to do the kind of analysis done in with the original speeds?

No. We need to only change the function definition. As time permits, use desmos.com to investigate.

