

### Objectives for Functions Activity

- Determine whether or not a given rule is a function
- Use and interpret function notation
- Determine domain and range of a function
- Evaluate a function at an input
- Given an output to a function, find its input
- Combinations of functions
- Piecewise defined function
- Interpret the zeros of a function
- Identify where a function is increasing, decreasing, constant

Determine whether or not the following situations are functions. Give an explanation for your answer.

Input	Output
1	2
3	3
5	2
7	3
9	2

Yes/No:

Explain:

Input	Output
a	1
b	2
c	3
d	4
e	5

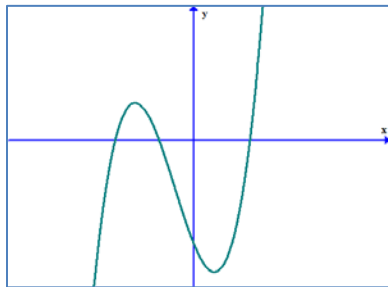
Yes/No:

Explain:

Input	Output
4	1
8	3
9	7
1	3
8	10

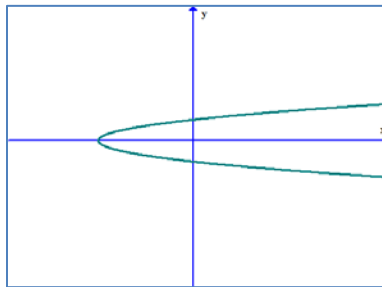
Yes/No:

Explain:



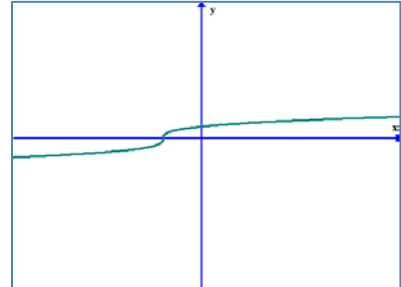
Yes/No:

Explain:



Yes/No:

Explain:



Yes/No:

Explain:

Input: Social Security Number  
Output: Person

Yes/No:

Explain:

Input: Model of a Vehicle  
Output: Manufacturer

Yes/No:

Explain:

Input: Phone Number  
Output: Person

Yes/No:

Explain:

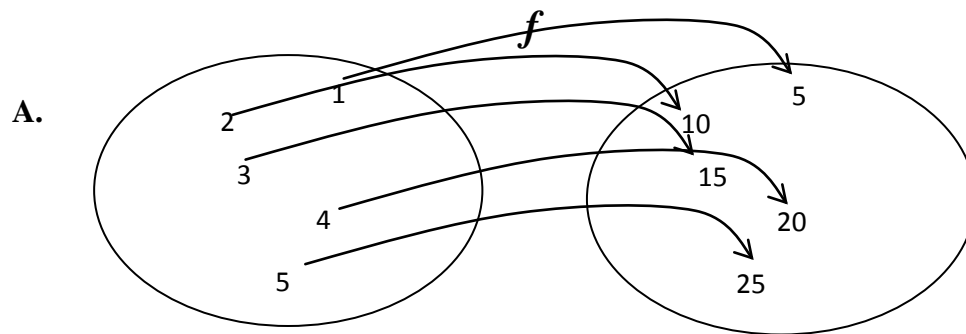
## Function Notation

$$y = f(x)$$

**output =  $f$  (input)**

1. Use function notation to write  $y$  as a function of  $x$ .
2. Now use function notation to write  $m$  as a function of  $v$ .
3. The number of calories used per minute depends on body weight.
  - a. Use function notation to write number of calories,  $c$ , as a function of body weight in pounds,  $p$ .
  - b. Name the *independent* variable: \_\_\_\_\_  
Name the *dependent* variable: \_\_\_\_\_
4. Consider the hypothetical situation where a car is traveling at a constant speed of 60 mph. The total distance that the car travels depends on the amount of time it travels.
  - a. If  $d$  represents distance in miles and  $t$  represents time in hours, write a formula for  $d$  as a function of  $t$ . Use function notation.
  - b. Which is the *input* variable? \_\_\_\_\_  
Which is the *output* variable? \_\_\_\_\_
5. Suppose  $T = f(c)$ . Which letter represents the *independent* variable? the *dependent* variable? *input* variable? *output* variable? Which letter represents the name of the function?

## Working with Function Notation



1.  $f(1) = ?$
2.  $f(3) = ?$
3.  $25 = f(x)$ . What is the value of  $x$ ?
4.  $10 = f(x)$ . What is the value of  $x$ ?

## B. $h(\text{input}) = \text{output}$

Inputs	Outputs
-2	-8
-1	-5
0	9
1	0
2	-3
3	-10

1.  $h(-1) = ?$
2.  $h(0) = ?$
3.  $-3 = h(x)$ . What is the value of  $x$ ?
4.  $0 = h(x)$ . What is the value of  $x$ ?

C.  $g: \{(1, 2), (3, 4), (5, 6), (7, 8), (9, 6), (11, -3)\}$

1.  $g(5) = ?$

2.  $g(11) = ?$

3.  $4 = g(x)$ . What is the value of  $x$ ?

4.  $6 = g(x)$ . What is the value of  $x$ ?

The function,  $f$  gives the revenue  $R$ , of a company(measured in 1000s of dollars) for producing  $x$  units of a product. Write a sentence to describe the following:

$$f(540) = 22.34$$

## What is a Function?

For each statement below, circle True or False. If you think the statement is False, write a statement that makes it True.

True   False   A function is a **rule** which takes certain values as inputs and assigns to each input value exactly one output value. The output is a function of the input.

True   False   A function is a **relationship** between 2 quantities. If the value of the first quantity determines exactly one value of the second quantity, we say the second quantity is a function of the first.

True   False   A function tells the relationship between the **independent** variable (**input**) and the **dependent** variable (**output**).

True   False   Functions don't have to be defined by formulas alone. We can use tables, graphs and words to define functions.

True   False   Not all relations are functions.

True   False   The Vertical Line Test tells you whether a graph is a function.

### Practice Using Function Notation

For problems 1-6, use the linear function  $f(x) = 3x + 2$ .

1. Calculate  $\frac{f(4) - f(1)}{4 - 1}$ :

2. Find  $f(a)$ :

3. Find  $f(a + h)$ :

4. Find  $f(a + h) - f(a)$  and simplify:

5. Find  $f(x + 1)$  and simplify:

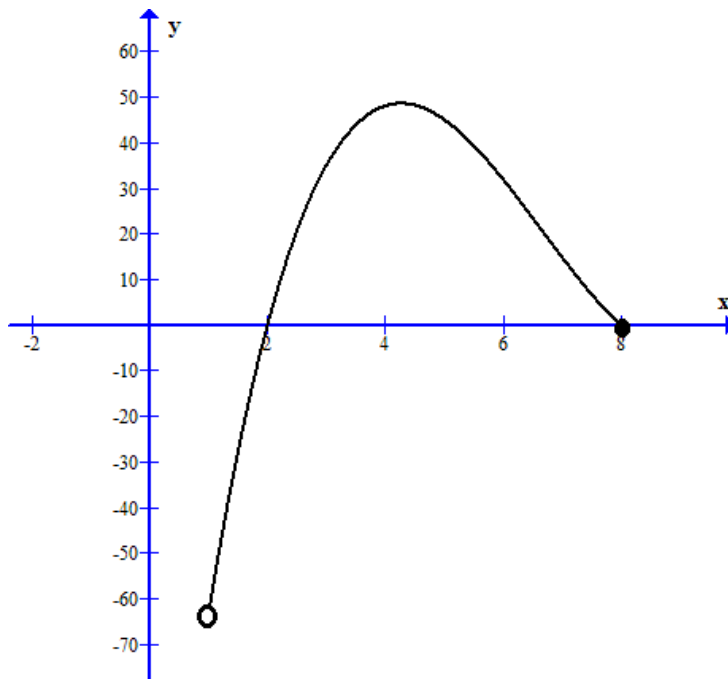
6. Find  $f(x + 1) - f(x)$  and simplify:

## Domain and Range

The domain of a function is the set of all inputs that yield an output. The range is the set of corresponding outputs.

Give the domain and range for the following function:

Use interval notation



Domain:

Range:

Find the domains of the following functions:

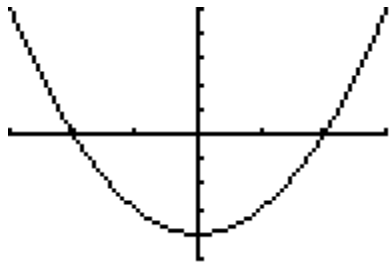
$$f(x) = 3x - 5$$

$$g(x) = \frac{3x - 4}{2x + 1}$$

$$h(x) = \sqrt{3x + 5}$$



$$f(x) = x^2 - 4$$

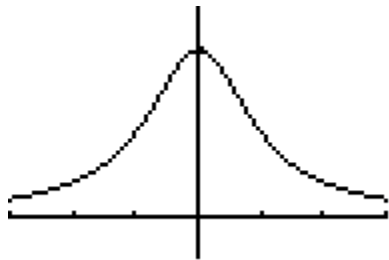


```
WINDOW
Xmin=-3
Xmax=3
Xscl=1
Ymin=-5
Ymax=5
Yscl=1
Xres=1
```

As the *domain* values vary from -3 to 0, the *range* values vary from \_\_\_\_ to \_\_\_\_.

As the *domain* values vary from 0 to 3, the *range* values vary from \_\_\_\_ to \_\_\_\_.

$$f(x) = \frac{1}{x^2 + 1}$$

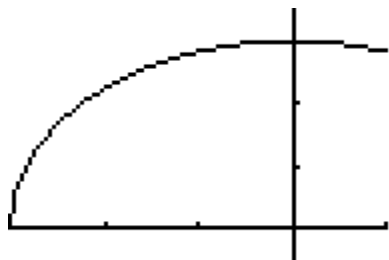


```
WINDOW
Xmin=-3
Xmax=3
Xscl=1
Ymin=-.25
Ymax=1.25
Yscl=1
Xres=1
```

As the *domain* values vary from -3 to 0, the *range* values vary from \_\_\_\_ to \_\_\_\_.

As the *domain* values vary from 0 to 3, the *range* values vary from \_\_\_\_ to \_\_\_\_.

$$f(x) = \sqrt{x + 3}$$



```
WINDOW
Xmin=-3
Xmax=1
Xscl=1
Ymin=-.5
Ymax=3.5
Yscl=1
Xres=1
```

As the *domain* values vary from -3 to 0, the *range* values vary from \_\_\_\_ to \_\_\_\_.

As the *domain* values vary from 0 to 1, the *range* values vary from \_\_\_\_ to \_\_\_\_.

### Combinations of Functions

Given  $f(x) = x^2 + 2x - 3$  and  $g(x) = -x^2 + 5x - 5$ , find the following:

1.  $f(x) + g(x)$

2.  $f(x) - g(x)$

3.  $2f(x) + 3g(x)$

4.  $f(2) - g(4)$

5.  $f(x) \cdot g(x)$

6.  $f(x) / g(x)$

## Piece-Wise Defined Functions

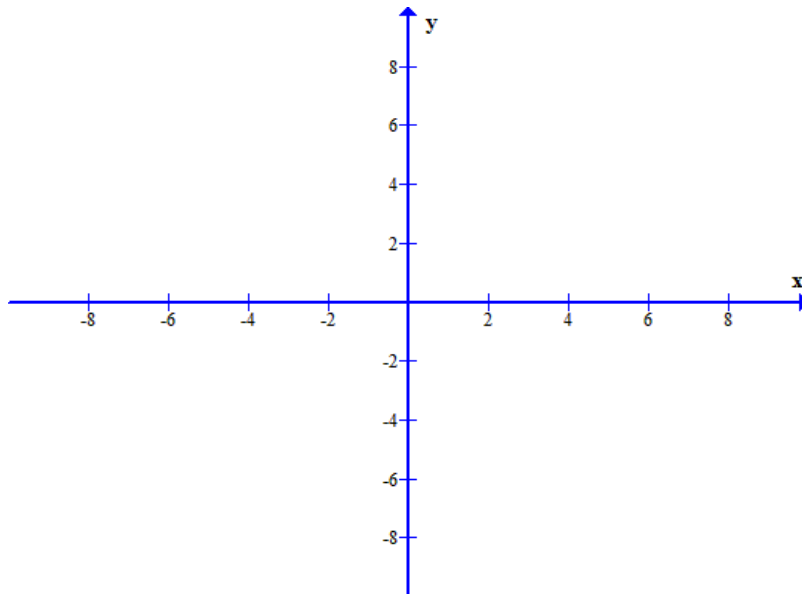
Given the following piecewise defined function, answer the questions.

$$f(x) = \begin{cases} x^2 - 1, & x < -1 \\ 0, & -1 \leq x < 2 \\ 4x - 4, & x \geq 2 \end{cases}$$

Calculate the following values:

1.  $f(-3) =$
2.  $f(-1) =$
3.  $f(0) =$
4.  $f(2) =$
5.  $f(3) =$

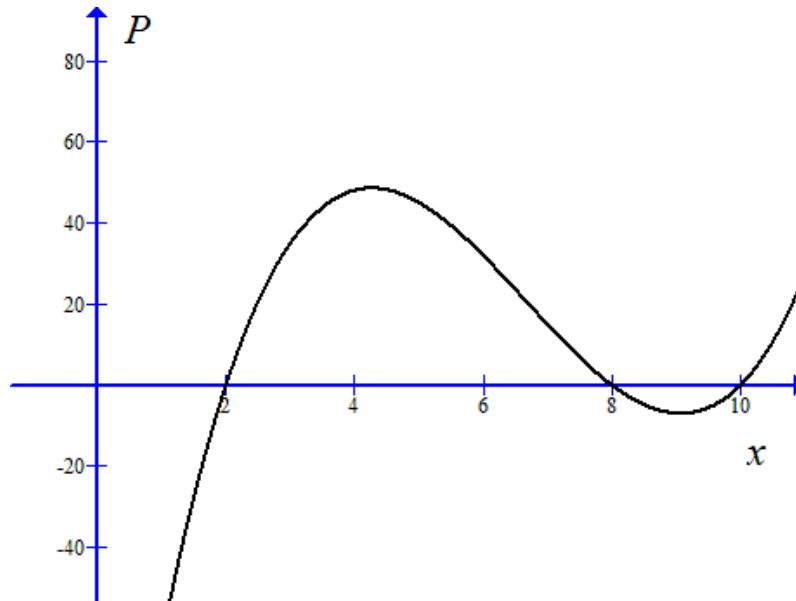
Draw a sketch of the graph



## Zeros of a Function

The zeros of a function are the inputs that make the outputs 0.

The following graph displays a company's profit (in thousands of dollars) as a function of the number of items produced (measured in hundreds of items).

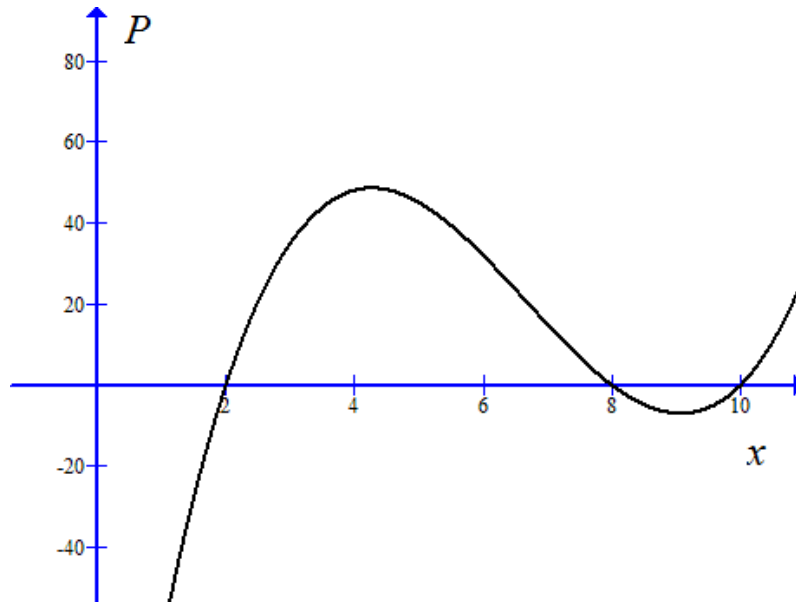


Find the zeros and interpret them in the context of the situation.

## Behavior of a Function

A function is increasing if its outputs increase as its inputs increase. A function is decreasing if the outputs decrease as the inputs increase.

Using the following graph, determine where the function is increasing and where it is decreasing. Put your answer in interval notation.



Increasing:

Decreasing:

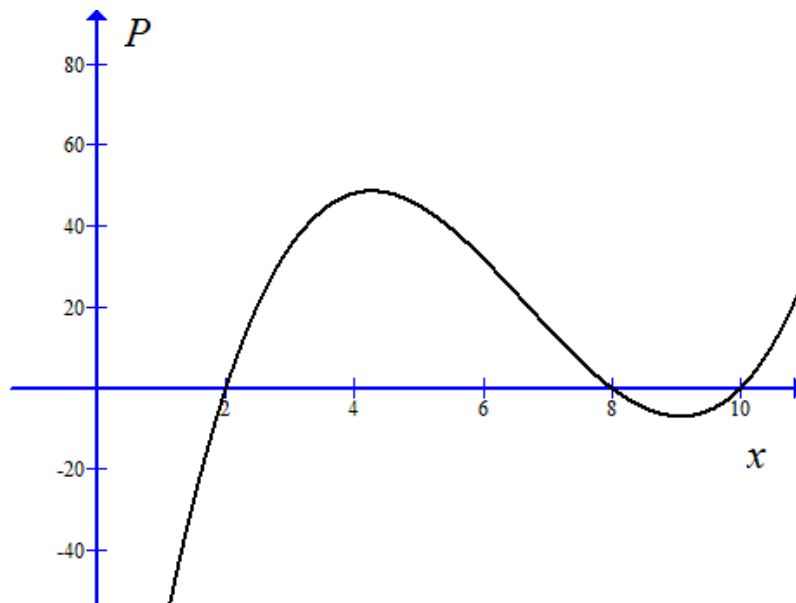
**Reflection:** Write 2-3 sentences to explain how you can tell from a graph whether a function is *increasing* or *decreasing*.

## Relative and Absolute Extrema

We have a relative minimum if a function has a point that is less than all of the points around it. This often occurs when a function goes from decreasing to increasing. We have a relative maximum if a function has a point that is greater than all of the points around it. This often occurs when a function goes from increasing to decreasing.

The maximum value (sometimes referred to as the Absolute Maximum) of a function is the largest output value of a function over its entire domain. The minimum value (sometimes referred to as the Absolute Minimum) of a function is the smallest output value of a function over its entire domain.

Identify the relative extrema of the following function:



Relative minimum

Relative maximum

**Reflection:** Write 2-3 sentences to explain whether or not the relative extrema are the absolute extrema.

# Make It Real Book of Mathematical Models

## Custom Greeting Card Costs (Piecewise)

Source: [www.simplyfreshdesigns.com](http://www.simplyfreshdesigns.com)

Custom Greeting Card Costs		<div>Scatter Plot and Model Graph</div> <table><thead><tr><th>Number of Cards</th><th>Cost (dollars)</th></tr></thead><tbody><tr><td>25</td><td>31</td></tr><tr><td>30</td><td>36</td></tr><tr><td>35</td><td>41</td></tr><tr><td>40</td><td>46</td></tr><tr><td>45</td><td>51</td></tr><tr><td>50</td><td>56</td></tr><tr><td>50</td><td>44</td></tr><tr><td>60</td><td>51</td></tr><tr><td>70</td><td>58</td></tr><tr><td>80</td><td>65</td></tr><tr><td>90</td><td>72</td></tr><tr><td>100</td><td>79</td></tr></tbody></table>	Number of Cards	Cost (dollars)	25	31	30	36	35	41	40	46	45	51	50	56	50	44	60	51	70	58	80	65	90	72	100	79
Number of Cards	Cost (dollars)																											
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60	51																											
70	58																											
80	65																											
90	72																											
100	79																											
Design, Print, & Ship																												
\$1.00/card	white $4\frac{3}{4} \times 6\frac{1}{2}$																											
\$.75/card for orders of 50 or more	envelope included																											
(25 minimum order)																												
\$6 shipping fee on all orders																												
Source: <a href="http://www.simplyfreshdesigns.com">www.simplyfreshdesigns.com</a>																												

# Make It Real Book of Mathematical Models

## Asparagus Plant Cost (Piecewise)

Source: [www.henryfields.com](http://www.henryfields.com)

Jersey Supreme Hybrid Asparagus Plant Costs (2007 Catalog)		<div>Scatter Plot and Model Graph</div> <table><thead><tr><th>Asparagus plants</th><th>Total Cost (dollars)</th></tr></thead><tbody><tr><td>0</td><td>0</td></tr><tr><td>10</td><td>9.95</td></tr><tr><td>20</td><td>18.5</td></tr><tr><td>30</td><td>27.75</td></tr><tr><td>40</td><td>35</td></tr><tr><td>50</td><td>43.75</td></tr><tr><td>60</td><td>52.5</td></tr></tbody></table>	Asparagus plants	Total Cost (dollars)	0	0	10	9.95	20	18.5	30	27.75	40	35	50	43.75	60	52.5
Asparagus plants	Total Cost (dollars)																	
0	0																	
10	9.95																	
20	18.5																	
30	27.75																	
40	35																	
50	43.75																	
60	52.5																	
Plants	Cost per plant (dollars)																	
10	0.995																	
20, 30	0.925																	
40 or more	0.875																	