Proportional Relationships of Triangles
Lesson Plan

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Objectives:

- SWBAT demonstrate the proportional relationship between the corresponding sides and angles of right triangles.
- SWBAT solve problems using similar triangles.

During a unit on similarity and trigonometry, this lesson should be used ***after*** dealing with the similarity of triangles in general and learning the Pythagorean Theorem. This activity is great for an investigation into the special case of right triangle similarity.

Prologue:
This is a two-part activity and will most likely take two 50 - 55 minute class periods – one day per part. Part I (Day one) is a hands-on activity that allows students to work together on computers to discover the proportional relationship between a pair of similar right triangles. Ideally, you will have a class set of computers or a computer lab you could use for this lesson. If you don't have access to these resources you can run a demonstration on one computer and project it for the class and have students come up to manipulate the triangles.

In Part II (Day two) a situation is posed in which students will need to be able to use what they learned about similar right triangles in order to answer the questions.

Part I - Computer Investigation:
To start, have each student go to the website: http://tedcoe.com/math/?page_id=41. Keep in mind that the overall goal of this activity is for students to discover the proportional relationship between the corresponding sides of similar triangles. Consider having the students do the following to help direct them along the most fruitful path of discovery:

1) With the "adjust scale" slider set at s = 1, move the red and purple sliders. What do you notice that each of these do to the triangle in terms of the three sides and three angles? Be specific.
2) What do you think the blue-segmented line is provided for? Explain.
3) Now, change the "adjust scale" slider to s = 0.5. Describe what is different about what shows up on your screen.
4) With the s = 0.5 setting still intact, move the "adjust red" slider to where it is 2 blue segments long. Compare the length of the red side of the bigger triangle to the gold side of the smaller triangle. What do you notice about the location of each of these two sides? Choose from corresponding, opposite, or hypotenuse.
5) With the s = 0.5 setting still intact, move the "adjust red" slider to where it is first 8 blue segments long and then 7. (Note that you probably won't be able to see the full triangles in the windows provided on the website so be sure to use the red and blue segment measuring units to answer.) Record and then compare the length of the red side of the bigger triangle to the gold side of the smaller triangle in each case. What do you notice about the location of each of these two sides? What do you notice about the ratio between these sides?

6) Now study the three angles as the red slider is changed. What is true about the angles?
7) Finally, set the "adjust scale" slider to s = 1.4. Has anything changed with respect to the two triangles and their corresponding sides and angles? Explain.
8) How do you know that the smaller and the larger triangles are similar? Cite one of the theorems for triangle similarity.
9) What conjecture could be made about the corresponding sides of similar triangles from the data we collected about the red and gold triangle sides?
10) Does it appear that the other two sides of each triangle behave the same way? Give some evidence from the applet to either support of contradict this conjecture.

NOTE: Be sure to leave enough time to pull the lesson together and emphasize the major learnings that should have occurred.

<u>Part 2 - Act One</u>:
Dr. Cox, while visiting Maui on vacation, was fascinated by a palm tree growing strangely out of the side of a sand bank (see photo). Due to the tree being positioned at such an odd angle, he wondered about the length of the tree and its height off the ground. Of course, he didn't want to make a fool of himself by crawling out on the tree and dropping a tape measure or injuring himself by falling off in the process!

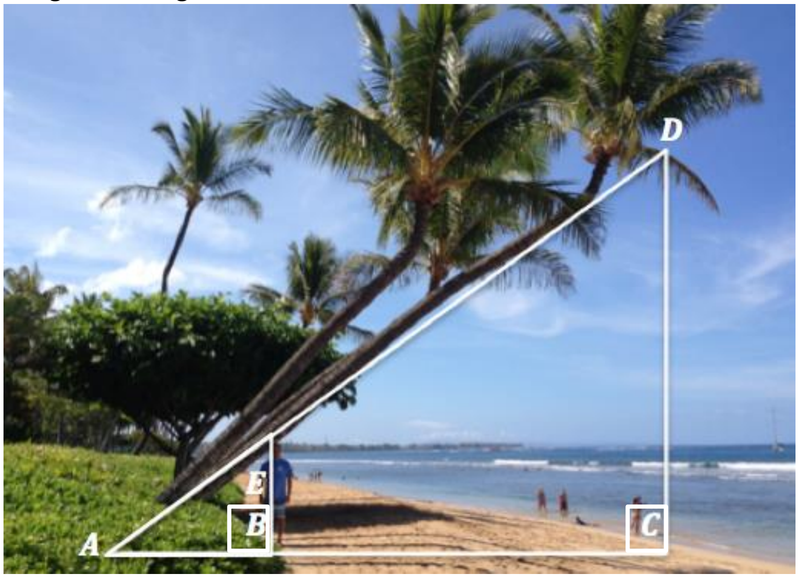
- Do you believe that it is possible to accurately estimate the length and height of the palm tree's trunk?
- Why or why not? Explain your thinking.

Trey Cox

<u>Act Two</u>:
More questions for you to consider:

- What information is important?
- What information doesn't matter?

Image - the angles



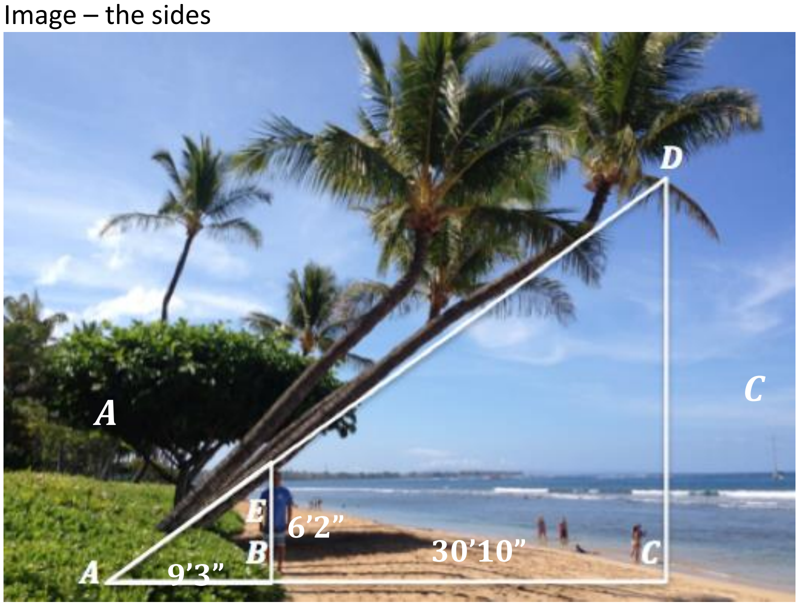
$\angle A \cong \angle A$ same angle in each triangle

$\angle B \cong \angle B$ both right angles

$\angle C \cong \angle C$ since both of the other angles are congruent

$\triangle ABE \sim \triangle ACD$

Image – the sides



$mBE = 6'2''$ Dr. Cox's height

$mAB = 9'3''$ Dr. Cox's distance from the base of the palm tree

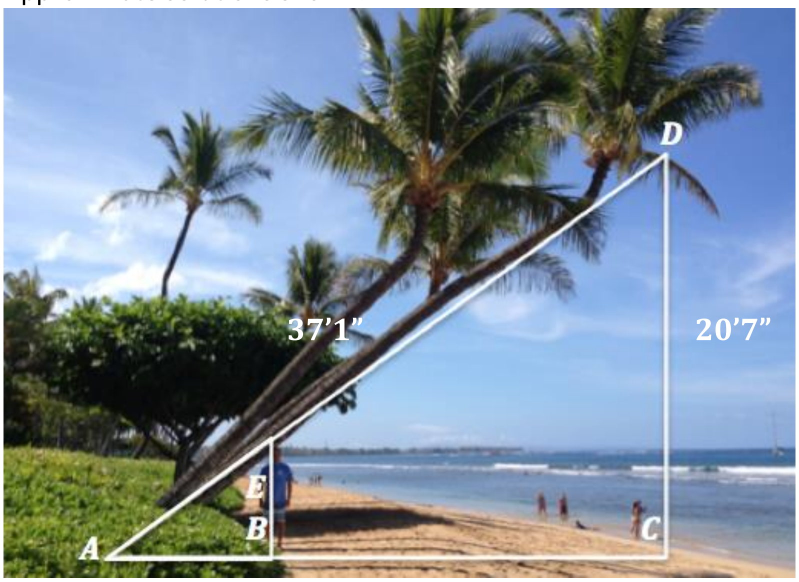
$mAC = 30'10''$ Length from the base to directly under the trunk

- Come up with an equation that will use the information that matters and then return a number that you can use to estimate the length of the tree.

- Come up with an equation that will use the information that matters and then return a number that you can use to estimate the height of the tree.

<u>Act Three</u>:

Image – the answer
Approximate solutions shown.



$$74^2 + 111^2 = (AE)^2$$
$$AE \approx 133.41''$$

$$\frac{111''}{370''} = \frac{74''}{DC}$$
$$DC \approx 246.67'' \approx 20'7''$$

$$\frac{DA}{133.41''} = \frac{370''}{111''}$$
$$DA \approx 37'1''$$

<u>Sequel I</u>:
Show how you could incorporate the trig function(s) into the solution of the prior problem.

<u>Sequel II:</u>
On the same vacation, Dr. Cox looked out off Kaanapali Beach in Maui and saw people having a wonderful time parasailing over the ocean. He read online that to parasail you must decide if you want to do "The 800" or "The 1200". These numbers refer to the "line length" in feet and not how high off the water you are. Use the images to estimate how high each of the parasailers would be if they were a) on "The 800" or b) on "The 1200".

Work here:

Work here:

Work here:

Work here:

Work here:

Work here: