Lesson Title:
An Introduction to Directed Distance – Rene's Folktale

Author: Trey Cox, Ph.D; Chandler-Gilbert Community College; Chandler, AZ; trey.cox@cgc.edu

Grade Level:

5 – 12

Overview:

This annotated lesson can be used to introduce ***directed distance*** and the concept of ***graph***. It can be used as the very first experience students have with graphs, as a review, and/or as an introduction to "circular" coordinates (you can choose to never refer to them as polar coordinates). It is highly interactive and connects the concepts of "new" graphing systems to rectangular coordinates. Initially, there is a brief history given and review of the Cartesian rectangular coordinate system. Before introducing other coordinate systems, students are then given the opportunity to "invent" their own graphing systems in teams and share them with the class. Without even telling students how to use the "circular" (polar) system, a natural segue is to let them explore the possibilities of how to use the circular polar grid on their own. [Note: Frequently students will develop the same or a very similar system on their own without having ever been introduced to the system known as polar coordinates.] The lesson facilitator may choose to then pull together the thoughts of students to lay out the conventional concepts of the polar coordinate system and formal (r,θ) notation. However, this is optional as the primary goal of the lesson is to give students additional insight into what it means to graph and the benefits of using the rectangular coordinate system.

For more advanced students or high school students, an overview of polar coordinates and lessons on how to convert from Cartesian to polar coordinates and vice-versa, Khan Academy YouTube video links are provided.

At the end of the lesson, an extension or honor's project is suggested that students investigate 3D graphing systems such as celestial, spherical, and cylindrical coordinates.

Purpose:

A purpose of this module is to help students recognize that there is a wide variety of different graphing systems of which rectangular and "circular" (polar) coordinates are only two possibilities. Secondly, the purpose is to give the students an intuitive understanding of the polar system. The teacher can choose to then present the students with the conventional knowledge and notation.

Prior Understandings:

Students need no prior knowledge of directed distance or graphs for this lesson. In fact, this lesson is designed to be the introductory activity for learning the concepts behind coordinate systems. It may be helpful for students to have a general understanding of:

- Quadrants
- Origin
- Ordered pair
- Rectangular coordinates (x = horizontal distance, y = vertical distance)

Materials:

- Geometric Coordinate Systems packet

Objectives:

Students will learn:

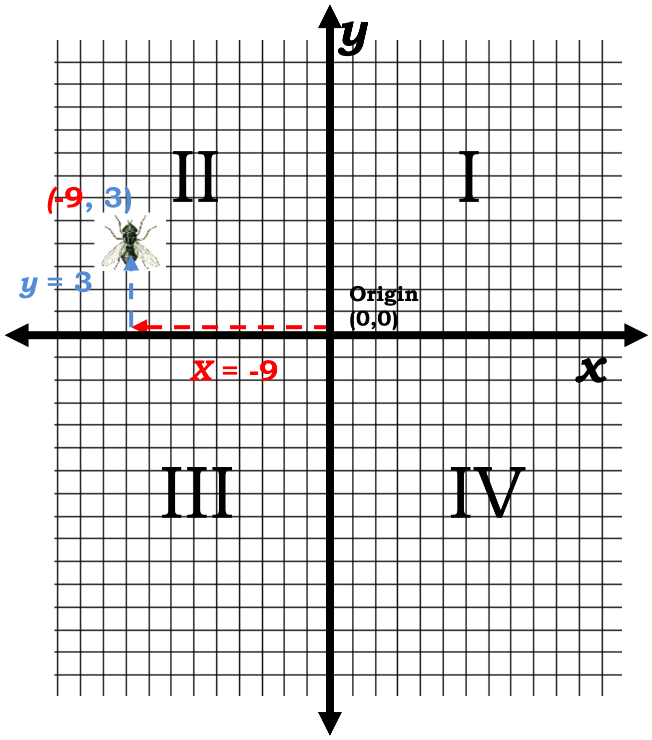
- Directed distance can be located with a variety of coordinate systems
- Rectangular and "circular" (polar) coordinates are related
- Similarities and differences between systems
- Strengths and weaknesses of a coordinate system
- Concepts behind the "circular" (polar) polar coordinate system and (*optional) how to convert from rectangular to "circular" (polar) polar and vice-versa

I. Instructor's Notes to: An Introduction to Directed Distance – Rene's Folktale

A. Brief History and Review of Cartesian Rectangular Coordinates:

Rene Descartes, is a famous philosopher who lived in France. During the time he was alive (1596 – 1650), much exploration was occurring including space with Galileo looking at planet orbits and distant lands were being discovered such as the New World of America with the Puritans. With such importance being placed on finding accurately locating points in the plane or space, Descartes turned his attention to this vexing problem.

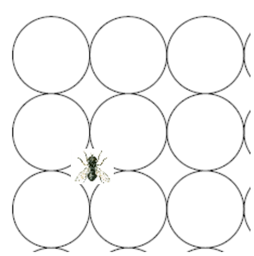
There is a well-known fable about Descartes and his invention of the rectangular coordinate system. Growing up Descartes was a sickly youth so his parents allowed him to stay in bed as long as he liked each day and into adulthood he continued this habit (a fact). The story is that one day while lying in bed he looked up at the ceiling and noticed that a fly had landed there. Wondering how he could locate that fly (a point) on the ceiling for someone arriving in his bedroom without simply pointing at it, he envisioned dividing the ceiling with two lines (the *x and y axes*) into what he called the *quadrants* I, II, III, and IV. He then decided that he could locate every point on the ceiling by starting from the point he called the *origin* using and *ordered pair* (0,0) to denote it. The first coordinate of the ordered pair he would use to measure horizontal distance and the second coordinate vertical distance. In the picture below we can see how the fly has “landed” at (-9, 3) and how this notation is interpreted as 9 units left of the origin and 3 units up.



NOTE: As we progress through this lesson, it is very important to make sure that students know and stay attuned to the fact that a graph is a collection of an infinite number of points that have two coordinates and that with Cartesian coordinates they measure horizontal and vertical distances.

B. Student Exploration:

Divide your classroom into working groups of 2 to 4. Provide each team one of the Geometric Coordinate Systems from the packet being sure to distribute a wide variety (***BUT DO NOT USE THE “CIRCULAR” (POLAR) COORDINATE SYSTEM***). Tell the students to imagine that their bedroom ceiling were designed with the pattern shown on the paper they were given. Ask them to consider how they would go about locating a “fly on the ceiling”. Through this activity students are encouraged to be creative and “invent” their own graphing systems either utilizing characteristics of Descartes’ system or coming up with new and unique ideas. Remind the student that the fly can land ANYWHERE on their coordinate system – even in any “open” spaces such as between circles as shown here.



If the students don't fully understand the task, have one partner highlight a spot on his/her paper and then try to get the other partner to highlight the exact same spot using only oral instructions. Or, consider challenging students to consider whether their system/instructions might change if they were trying to locate a particular triangle/circle/box vs. locate a particular intersection point (e.g. what if someone said "the fly is inside the second diamond from the left in the fifth row from the top" rather than identifying a specific point?)

After providing the teams adequate time to come up with a system they feel good about, choose individuals to present their work and thinking to the class. Encourage each presenter to share how their system works to locate "flies on the ceiling" and what they would perceive to be pros and cons to their newly created coordinate system. Also ask them how their system compares to Descartes' coordinate system.

As a whole class discussion, ask them to consider what the challenges were in terms of the various coordinate systems. Also have them discuss why they think Descartes' rectangular coordinate system has endured for almost 400 years?

C. Directed Distance Using Circles – Circular (Polar) Coordinates:

Continuing in their small groups, tell them you have one more Geometric Coordinate System to play with. Give them the "circular" (polar) grid in the packet but DO NOT give them any information about it or call it any special name. Without even telling students how to use the system, this activity will be a natural segue to let them explore the possibilities of how to use the circular grid on their own.

After adequate time for exploration and brainstorming, chose a few students to present their work to the class pulling out interesting ideas about how to use the system. Don't be surprised if they come up with the polar coordinate methods on their own.

You may choose to conclude the lesson by pulling together the thoughts of students and use it to lay out the conventional concepts of the polar coordinate system and formal (r,θ) notation.

You can provide the following links to the students for further review and study:

Khan Academy – Part 1 (Introduction to polar coordinates):
http://www.youtube.com/watch?v=jexMSlSDubM
Part 2 (Conversion from Cartesian to Polar Coordinates) -
http://www.youtube.com/watch?v=zGpbSGj_vfE
Part 3 (Converting between Cartesian and Polar functions) -
http://www.youtube.com/watch?v=9iqN12hCn10

D. Extension – Directed Distance:

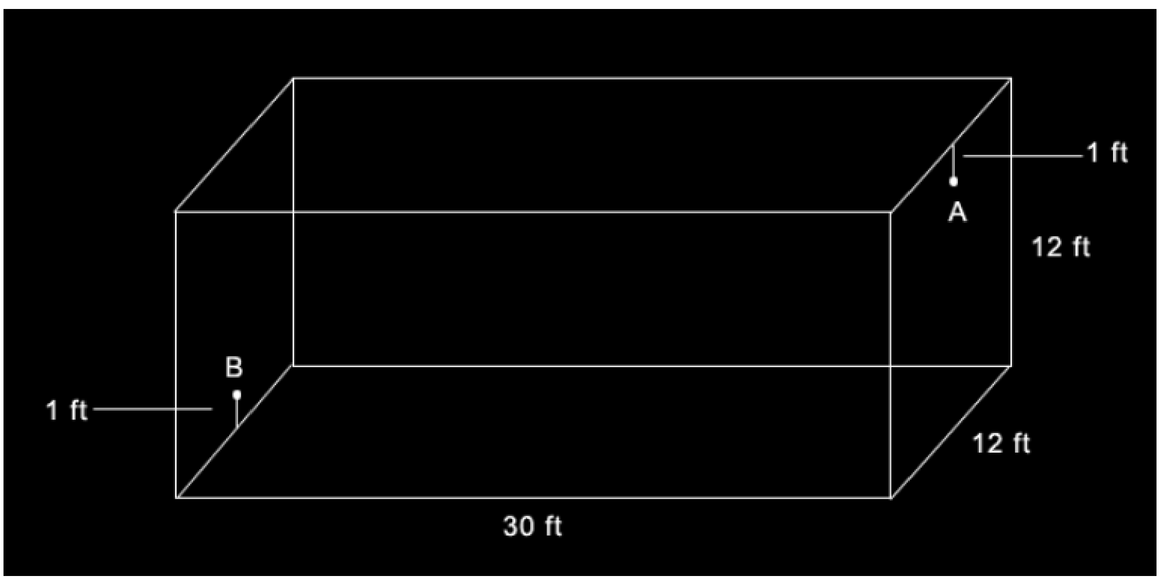
Option 1 - Extension to 3D: To extend the notion of directed distance, have students research celestial, spherical, and cylindrical coordinates online and report back the class.

Option 2 – Challenge problem:
This problem requires creativity and using the notion of similarity and the Pythagorean theorem.

Spider on the Wall

Henry Ernest Dudeney (1857-1930) is generally regarded as England's greatest creator of mathematical puzzles. The following puzzle is reproduced verbatim from his book "The Puzzles".

Inside a rectangular room, measuring 30 feet in length and 12 feet in width and height, a spider is at a point on the middle of one of the end walls, 1 foot from the ceiling, as at A; and a fly is on the opposite wall, 1 foot from the floor in the centre, as shown at B. What is the shortest distance that the spider must crawl in order to reach the fly, which remains stationary? Of course the spider never drops or uses its web, but crawls fairly.

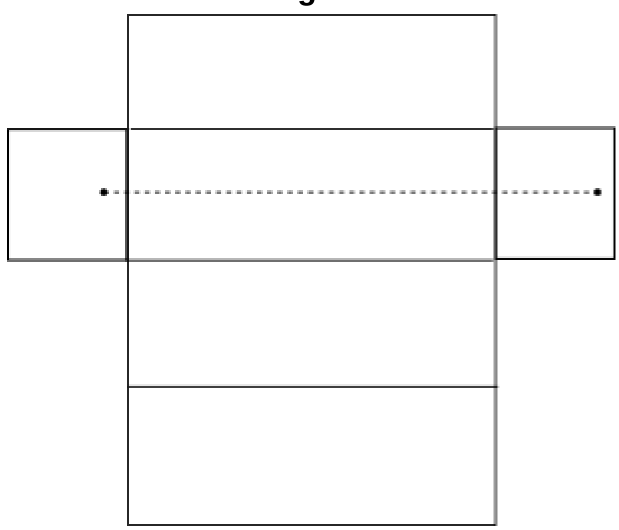


Solution:

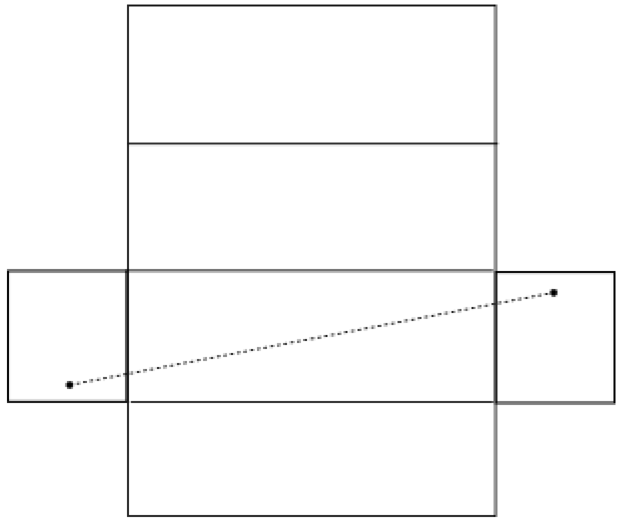
Let's start by considering a simpler problem. Imagine that the spider and the fly are on adjacent walls, and say that the spider wants to walk along the walls of the room to the location of the fly, along the shortest possible path. What would that path be? We could find the shortest path by unfolding the two walls, and then drawing a straight line between the spider and the fly.

The way to solve the original problem is analogous to this easier problem. We can treat the room as a rectangular prism, unfold it, and draw a line between the spider and the fly.

It's slightly more complicated this time, as we can unfold the prism in many ways. Since each end wall can be attached to one of four walls (ceiling, floor, long wall #1 and long wall #2), there are sixteen different ways to unfold the walls. However, because of the symmetry in the problem, we need only consider four of them. We can then use the Pythagorean theorem to figure out the shortest distance. Note that I've assumed below that the room is oriented so that the spider is on the east wall and the fly on the west wall just for the sake of simplifying the descriptions; of course, it doesn't matter how the room is oriented.

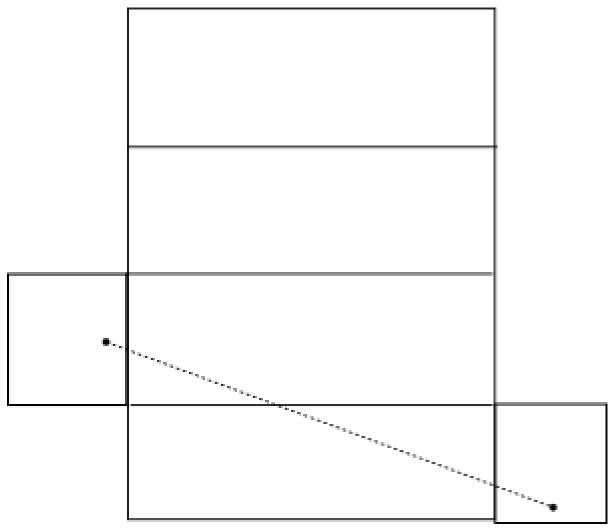
Description	Diagram	Distance
End walls hinged to floor		1 + 30 + 11 = 42

End walls hinged to north wall



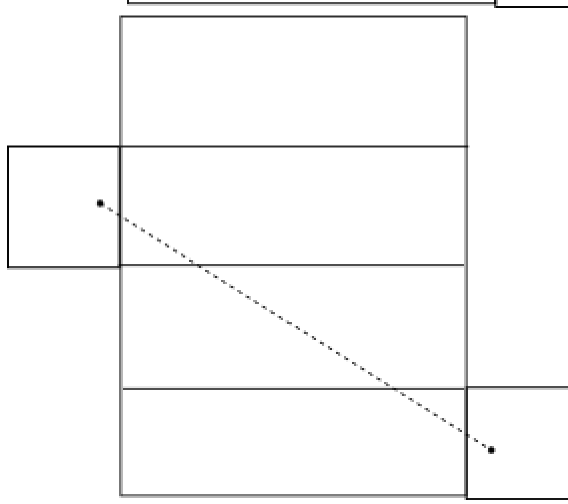
$\sqrt{(6 + 30 + 6)^2 + 10^2} \approx 43.2$

West wall hinged to floor, east wall hinged to south wall



$\sqrt{(1 + 30 + 6)^2 + (6 + 11)^2} \approx 40.7$

West wall hinged to floor, east wall hinged to ceiling



$\sqrt{(1 + 30 + 1)^2 + (6 + 12 + 6)^2} = 40$

The shortest result is the last one in the table. Perhaps this is a slightly unintuitive result, because the shortest path ends up passing through five of the six of the walls of the room, but that is the shortest solution.