Rule Time: Salute to Brakes

The following group project is to be worked on by *no more than four* students. You may use any materials you think may be useful in solving the problems but *you may not* ask anyone for help other than the people you have chosen to work together. This means you may not ask a tutor or any person other than those in your immediate group for help.

You are to type a letter of response to the problem presented backing up your conclusions with mathematical reasoning, formulas, and solutions. Your grade will depend on how well you communicate your response as well as the accuracy of the conclusions. This project will be scored on the checklist that is attached.

Please sign and date here to indicate that you have read and agree to abide by the above mentioned stipulations.

______________________________ _______________________
Student Name #1 Date

______________________________ _______________________
Student Name #2 Date

______________________________ _______________________
Student Name #3 Date

______________________________ _______________________
Student Name #4 Date

Rule Time: Salute to Brakes

July 1, 2001

Jim "The Rule Man" Baylor
1892 W. Springfield Way
Chandler, AZ 85248

Algebra Advisors
Chandler-Gilbert Community College
2626 East Pecos Road
Chandler, AZ 85225

Dear Algebra Advisors:

"Will Hal stop in time?"

"Is this the end of Rule Time?"

"Do the math and find out!"

I am Jim "The Rule Man" Baylor and I come to you in this, my time of dire need. The show "Rule Time" has become a very popular show. It seems that the public is interested in the role measurement plays in the world around them. Maybe you saw one of our past episodes - Salute to Sphygmomanometers or Salute to Sports. Or more recently, Salute to Speed or Salute to Flight. Yes, measurement is very important to me and it is my mission to spread the joy of accurate measurement to the world!

That brings us to my latest dilemma. I have found a better way to stop a moving car. You may have heard of ABS (Antilock Braking System), but I have created the JBS…Jim's Braking System…with more POWER! These brakes have the titanium, kryptonite, NASA approved, Velcro lined, flannel driven…ARR! ARR! ARR! ARR! I'm sorry! I just get so excited about the opportunity to measure the effectiveness of this newly invented braking system! But this is also what has gotten me into a bit of a spot.

Hal (Flannel Man) and I have solicited the help of our friends at the GM Proving Grounds in order to accurately measure the effectiveness of the JBS braking system. My friend Todd is the chief engineer out there and he has been gracious enough to let us film an episode of Rule Time in order to bring his professional work to our audience. The only problem is that he is so darn nit-picky. At the rate he is proceeding through this experiment, it will take hours to get the information I ultimately want my viewing public to see - the distance needed to stop a car going 100-MPH.

Todd has gathered stopping distance data up to 50-MPH and I think that I can use these data to project to 100-MPH. Here is the data that Todd has collected:

Speed (MPH)	Reaction Distance (feet)	Braking Distance (feet)
20	22.0	22.2
25	27.5	34.7
30	33.0	50.0
35	38.5	68.0
40	44.0	88.8
45	49.5	112.4
50	55.0	138.8

To me, it makes perfect sense to add the reaction distance to the braking distance for the 50-MPH speed and then double that to see what the stopping distance would be at 100 MPH. Wouldn't that make perfect sense? Apparently not to Todd and Hal! They seem to have another idea. And since Todd wouldn't help me with the experiment, I had to take matters into my own hands (did he call me a dummy?).

This is where you come in! I need you to work up a complete analysis of this situation and send it to me. I know that it will be too late to help me in my current situation since Hal has already started the experiment, but I would like to study your analysis so that I can be more safe next time!

Attached to this letter is a laboratory activity created by Hal "I thought engineers just drove trains" Morland. Please work through his activity before writing your letter to me.

I would appreciate hearing back from you by ___________________.

Sincerely,

The Rule Man

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Stopping Distance

Introduction

In this lesson you will apply your knowledge of functions to a real-world situation and to experience how functions can be used to model a situation. You will explore a situation regarding the stopping distance of an automobile, analyze the given data, and make a prediction based on the data. The use of technology to develop a mathematical model used in prediction will also be developed in this lesson.

Learning Objectives

- Solve a real life math problem involving multiple and sequential steps in order to answer a question.
- Use data to model a real-world situation.
- Represent data in three different formats: graphically, numerically (tables), and algebraically (symbolic).
- Interpret the data to answer questions posed about the situation.
- Determine the difference between data that represents a linear model and data that represents a quadratic model.
- Calculate a mathematical model to represent given data and use that model to make predictions and analyze the given situation.

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Notes to the Instructor:

Time: This lesson can be completed in two 50 or 75minute class periods.

Pre-requisites: Students need to have the experience of doing regression on the graphing calculator from Lesson 5.

Lesson Notes: This lesson allows students to apply their knowledge from this module to a practical situation. The lesson asks students to explore the data graphically and numerically first in order to predict the braking distance of a car travelling 100 MPH. At the end of the lesson, quadratic regression on the calculator is utilized to verify the prediction.

Homework: Students are expected to use the regression feature on their graphing calculator.

Hal Morland's Stopping Distance Activities

Activity 1: Stopping Distance - Graphical Exploration

The data below shows the distance it takes automobiles traveling at various speeds to come to a complete stop on dry, clean, level pavement. The ***reaction distance*** is the distance the automobile travels from the moment the driver decides to apply the brake to the moment the brake is applied. ***Braking distance*** is the distance traveled from the moment the driver applies the brake to the moment the auto comes to a complete stop.

Speed (MPH)	Reaction Distance (feet)	Braking Distance (feet)
20	22.0	22.2
25	27.5	34.7
30	33.0	50.0
35	38.5	68.0
40	44.0	88.8
45	49.5	112.4
50	55.0	138.8

Table 6.1 Reaction Distance and Braking Distance
(Source: Minnesota Driving Manual)

Our main goal in the next several activities is to predict a accurate stopping distance for a car traveling 100 MPH.

1. Create a new column that would represent what is called the ***stopping distance*** at each speed by adding the reaction distance and the braking distance.

Speed (MPH)	Stopping Distance
20	44.2
25	62.2
30	83.0
35	106.5
40	132.8
45	161.9
50	193.8

Table 6.2 Stopping Distance

2. Using the table of data above, estimate the Stopping Distance at each of the following speeds. Explain your answers and discuss which were easier to estimate and which were more challenging to estimate.
   a) 15 MPH
   b) 28 MPH
   c) 55 MPH
   d) 75 MPH
   e) 100 MPH

Note: Students are expected to interpolate based on the data. It is not expected that complex mathematics be done at this point in the lesson. Answers will vary, but students will probably think linearly.

3. The variable that is known, in this case the speed of the car is used as the independent variable and is plotted on the x-axis. The quantity to be measured, in this case the braking distance, is used as the dependent variable and is plotted on the y-axis.
   a) Plot the data representing Speed vs. Stopping Distance. Scale the grid to be [0,120] (MPH) by [0,600] (feet).

   b) Connect the points with a smooth curve.

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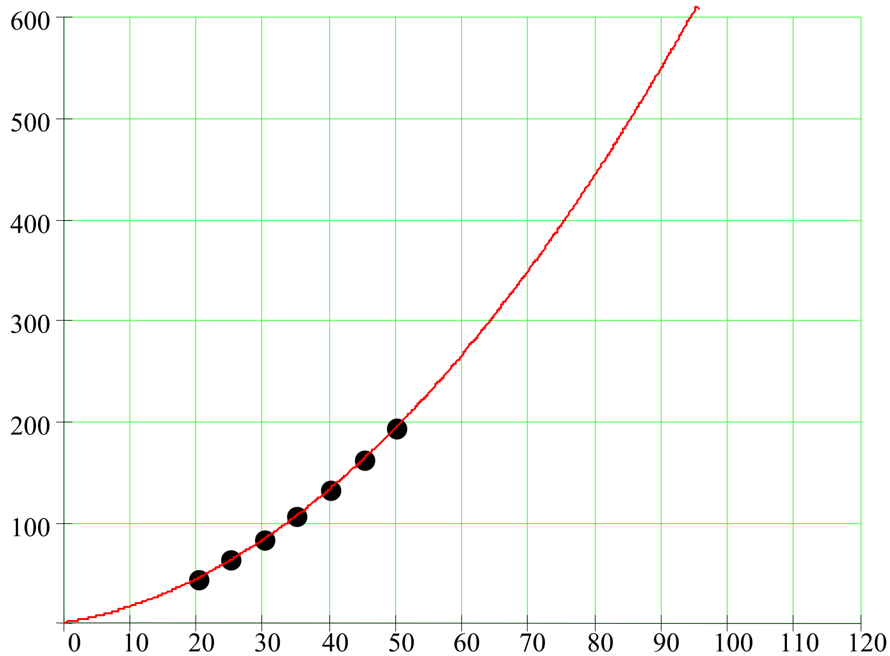
Figure



6.1

4.

a) Use the graph to verify or change your answers to question 2.

i) 15 MPH about 30 feet

ii) 28 MPH about 80 feet

iii) 55 MPH about 230 feet.

iv) 75 MPH about 400 feet

v) 100 MPH about 700 feet

b) Explain how the graph is used in estimating the answers to 4a.

Locate the appropriate speed on the x-axis. Follow a vertical line straight up to the curve, then move horizontally straight over to the y-axis and read the distance value

c) Why is it appropriate to have the horizontal axis represent the speed and the vertical axis represent the stopping distance in this situation?

Braking distance depends on speed. The speed is known (independent) and the stopping distance is to be measured (dependent).

5. What type of function would best represent this data? Justify your answer.
   Answers will vary, but some students may look at the plotted points and think that it looks linear in the viewing window provided. They may say that a straight line would pass through all data points. This will be explored further in Activity 2.

Activity 2: Stopping Distance – Regression Analysis

1. Use the regression feature on your graphing calculator to create a linear, quadratic, and exponential model for stopping distance. Write this equation in function notation using D(S). Which function best represents this situation? Why?
   $D(S) = 0.056S^2 + 1.100S + 0.00079$

2. Use each of the regression models to calculate the stopping distance at the following speeds. Write each problem using function notation.

   a) 60 MPH D(60) = 265.84 feet

   b) 75 MPH D(75) = 394.75 feet

   c) 88 MPH D(88) = 526.67 feet

   d) 93 MPH D(93) = 582.40 feet

3. Use the functions above to predict the stopping distance of a car traveling 100 MPH.

   D(100) = 665.10 feet

Activity 3: Stopping Distance – Numerical Exploration

1. Calculate the difference between successive Stopping Distances from your table.

Speed (MPH)	Stopping Distance
20	44.2
25	62.2
30	83.0
35	106.5
40	132.8
45	161.9
50	193.8

Table 6.3

Successive Differences
18.0
20.8
23.5
26.3
29.1
31.9

2. Describe what you notice. Draw a representation of this difference on the graph.
   The difference is not constant, so the data cannot represent a linear function. Linear data would be represented by a constant successive difference. This difference is called the first difference for the data.
3. Calculate the differences between the successive differences calculated in part 2.

Speed (MPH)	Stopping Distance
20	44.2
25	62.2
30	83.0
35	106.5
40	132.8
45	161.9
50	193.8

Table 6.4

Successive Differences	
18.0	2.8
20.8	2.7
23.5	2.8
26.3	2.8
29.1	2.8
31.9	

4. Describe what you notice.
   The second difference is nearly constant (similar enough!) so that a quadratic function would best model this data. So, even though it may look linear graphically, the data is showing that a quadratic model is best. This is called the second difference of the data.
5. Work backwards from the second differences to extend Table 6.5 to include speeds up to 100 MPH. Plot this data on the graph in Figure 6.1.

Speed (MPH)	Stopping Distance
20	44.2
25	62.2
30	83.0
35	106.5
40	132.8
45	161.9
50	193.8
55	228.5
60	266.0
65	306.3
70	349.4
75	395.3
80	444.0
85	495.5
90	549.8
95	606.9
100	666.8

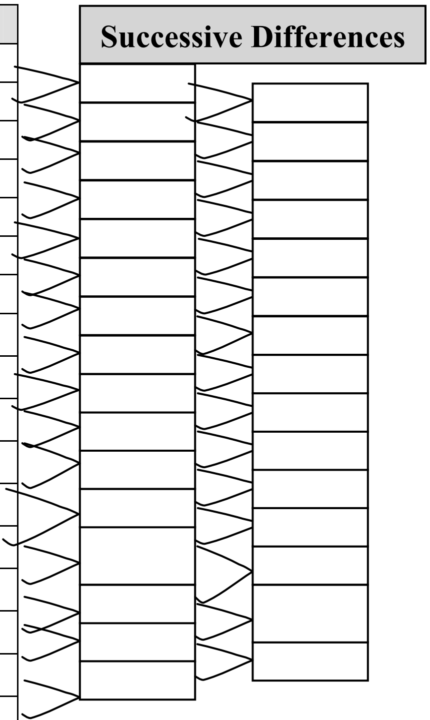


Table 6.5

6. Compare these numerical results with the regression models you used in Activity 2. Which model is consistent with your numerical findings?

7. Discuss with your group what it would mean if the first differences would have been all about the same. What do you think it means that the second differences are all about the same?

Activity 4: Stopping Distance – Prediction

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1. Use the graph in Figure 6-1 to estimate the Stopping Distance for an automobile traveling:

   a) 60 MPH 266 feet

   b) 75 MPH 395 feet

   c) 88 MPH 527 feet

   d) 93 MPH 582 feet

2. Use the graph to estimate the speed of an automobile that had a total braking distance of:

   a) 50 feet 20 MPH

   b) 300 feet 65 MPH

   c) 450 feet 80 MPH

   d) 600 feet 95 MPH

3. Provide a written explanation for how the graph you have created helps you make the previous estimations.

   First, the curve passing through the data points must be extended.
   In number 1, the appropriate speed is located on the x-axis, move vertically up to the curve. From that point, move horizontally to the y-axis and read the value.
   In number 2, find the appropriate distance on the y-axis and move

   horizontally over to the curve. From that point on the curve, move vertically down to the x-axis and read the value.

4. Explain any restrictions on the domain and range for the graph you created.

The domain is restricted by the speed of the car…maybe [0, 120]
The range then is restricted similarly…maybe [0,930]

5. Mathematicians have developed a notation for the previous estimations. To write the question "What is the Stopping for an automobile traveling 60 MPH", mathematicians would simply write D(60). If you would like to write the question more generically, you would write D(S).

   a) What does D represent? The Stopping Distance

   b) What does S represent? The Speed of the car.

   c) How do you say D(S)? D of S.

   d) Explain the meaning of D(S). It describes the Stopping Distance

6. Write the question and answer to 6 a) – d) using this notation.

   a) 60 MPH T(60)=266

   b) 75 MPH T(75)=395

   c) 88 MPH T(88)=527

   d) 93 MPH

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Writing Project Evaluation/Checklist

Gateway checklist - these items must be present in order for the paper to be evaluated

Yes	No	Expected Features
		1. Does this work meet the expectations for the presentation of technical work?
		2. Is the work all computer generated?
		3. Is there symbolic, numerical, and graphical support included in the work?
		4. Is the answer stated in a few complete sentences that stand on their own? That is, is the summary satisfactory?
		5. Is there a description of the solution(s)?
		6. Is the noise (i.e. grammatical, punctuation, spelling, etc. errors) level low enough to not cause communication problems?
		7. Is the project free of major errors?
		8. Is acknowledgment given where it is due, if appropriate?
		9. Is there an attached page describing the contributions of the team members?

Your final score will be calculated based on your performance on these features:

Very Good	Good	Poor	
			Clear summary of the problem to be solved • Introductory paragraph lays the background for the problem situation and its solution • Shows why the question(s) to be addressed are important
			Precise and well-organized explanation of how the answer was found including • assumptions • algebraic (symbolic) support • graphical support • numerical support

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Very Good	Good	Poor	Features
			Solve the problem(s) that were originally asked so that there are no obvious errors in the solution. Shows familiarity with the mathematical concepts and their appropriate use.
			Use of graph mechanics including • labeled axes with units • labeled axis divisions • descriptive title • clear and descriptive legend • data points shown
			Concluding paragraph summarizes the purpose of the project and the outcome. Briefly closes the letter by stating any limitations or suggestions for improvement.
			Style and readability demonstrates a quality of imagination and rigor that results in a distinctive project. The project shows a personal exploration.

Comments on quality of submitted work and how any problems might be resolved

Final Score:_______________________________

T(93)=582