S-CP The Titanic 2

Task

On April 15, 1912, the Titanic struck an iceberg and rapidly sank with only 710 of her 2,204 passengers and crew surviving. Some believe that the rescue procedures favored the wealthier first class passengers. Data on survival of passengers are summarized in the table below. We will use this data to investigate the validity of such claims. (Data source: http://www.encyclopedia-titanica.org/titanic-statistics.html)

	Survived	Did not survive	Total
First class passengers	201	123	324
Second class passengers	118	166	284
Third class passengers	181	528	709
Total passengers	500	817	1317

a. Are the events “passenger survived” and “passenger was in first class” independent events? Support your answer using appropriate probability calculations.

b. Are the events “passenger survived” and“passenger was in third class” independent events? Support your answer using appropriate probability calculations.

c. Did all passengers aboard the Titanic have the same probability of surviving? Support your answer using appropriate probability calculations.

IM Commentary

This is the second task in the series of three, which ask related questions, but use different levels of scaffolding. Also, the third task uses a more detailed version of the data table. This task lets students explore the concepts of probability as a fraction of outcomes, and using two-way tables of data. The special emphasis is on developing their understanding of conditional probability and independence. Students should be able to use conditional probability to decide if two events A and B are independent; that is they should check if $P(A|B) = P(A)$, or if $P(B|A) = P(B)$.

In the last part of the task students have to decide which probabilities would be useful to answer the given question, and calculate them.

This task could be used as a group activity where students cooperate to formulate a plan of how to answer each question and calculate the appropriate probabilities. The task could lead to extended class discussions about the different ways of using probability to justify general claims (i.e. Can we really say that first class passengers had a greater chance of being rescued? Why or why not?)

The other tasks in this series are S-CP.1,4,6 The Titanic 1 and S-CP.4,5,6 The Titanic 3.

Solution

a. We use the fact, that two events A and B are independent, if $P(A|B) = P(A)$. In this case, we compare the conditional probability
$P(\text{passenger survived}|\text{passenger was in first class})$ with the probability
$P(\text{passenger survived})$.

The probability of surviving, given that the passenger was in first class, is the fraction of first class passengers who survived. That is, we restrict the sample space to only first class passengers to obtain
$P(\text{passenger survived}|\text{passenger was in first class}) = \frac{201}{324} \approx 0.620.$

The probability that the passenger survived is the number of all passengers who survived divided by the total number of passengers, that is
$P(\text{passenger survived}) = \frac{500}{1317} \approx 0.380$. Since $0.620 \neq 0.380$, the two given events are not independent. Moreover, we can say that being a passenger in first class increased the chances of surviving.

Note, that we could also compare
$P(\text{passenger was in first class}|\text{passenger survived}) = \frac{201}{500} \approx 0.402$ and
$P(\text{passenger was in first class}) = \frac{324}{1317} \approx 0.246$. Again, since $0.402 \neq 0.246$, the two events are not independent.

b. Using similar reasoning as in part (a), we compare
$P(\text{passenger survived}|\text{passenger was in third class}) = \frac{181}{709} \approx 0.255$, and
$P(\text{passenger survived}) = \frac{500}{1317} \approx 0.380$. Since $0.255 \neq 0.380$, the two given events are not independent. Moreover, we can see that being a passenger in third class decreased the chances of being rescued.

c. One way to answer this question is to compare the probabilities of surviving for randomly chosen passengers in first, second, and third class, respectively. To do this, we calculate the following conditional probabilities:

- In part (a) we calculated that
  $P(\text{passenger survived}|\text{passenger was in first class}) = \frac{201}{324} \approx 0.620$.
- The probability that the passenger survived, given that this passenger was in second class, is the fraction of passengers in second class who survived, that is
  $P(\text{passenger survived}|\text{passenger was in second class}) = \frac{118}{284} \approx 0.415$.
- In part (b) we calculated that
  $P(\text{passenger survived}|\text{passenger was in third class}) = \frac{181}{709} \approx 0.255$.

Comparing these probabilities we can say that not all passengers aboard the Titanic had the same chance of surviving. More precisely, the chance of surviving depended on the class, with the first class passengers having the greatest, and the third class passengers having the smallest chance of being rescued.

Note that there are different probabilities we could use to answer this question (for example we could compare probability that a randomly selected passenger survived $P(\text{passenger survived}) = \frac{500}{1317} \approx 0.380$ with the conditional probability $P(\text{passenger survived}|\text{passenger was in first class}) = \frac{201}{324} \approx 0.620$). However, the conclusion should always be the same.