S-CP The Titanic 3

Task

On April 15, 1912, the Titanic struck an iceberg and rapidly sank with only 710 of her 2,204 passengers and crew surviving. Some believe that the rescue procedures favored the wealthier first class passengers. Other believe that the survival rates can be explained by the "women and children first" policy. Data on survival of passengers are summarized in the table below. Investigate what might and might not be concluded from the given data. (Data source: http://www.encyclopedia-titanica.org/titanic-statistics.html)

	Survived	Did not survive	Total
Children in first class	4	1	5
Women in first class	139	4	143
Men in first class	58	118	176
Children in second class	22	0	22
Women in second class	83	12	95
Men in second class	13	154	167
Children in third class	30	50	80
Women in third class	91	88	179
Men in third class	60	390	450

Total passengers	500	817	1317

IM Commentary

This is the last task in the series of three, which ask related questions, but use different levels of scaffolding. This task uses a more detailed version of the data table. This is a very open ended task. It poses the question, but the students have to formulate a plan to answer it, and use the two-way table of data to find all the necessary probabilities. The special emphasis is on developing their understanding of conditional probability and independence. This task could be used as a group activity where students cooperate to formulate a plan of how to answer the question and calculate the appropriate probabilities. The task could lead to extended class discussions about the different ways of using probability to justify general claims (i.e. Can we really say that first class passengers had a larger chance of being rescued? Why or why not? What was the role of gender in the rescue procedures?)

The other tasks in this series are S-CP.1,4,6 The Titanic 1 and S-CP.3,4,5,6 The Titanic 2.

Solution

Note that there are different ways we could answer this question. First, we ignore the gender and compare the probability of surviving for a randomly chosen passenger in first class, to the probabilities of surviving for randomly selected second and third class passengers, respectively. To do this, we calculate the following conditional probabilities.

- The probability that the passenger survived, given that the this passenger was in first class, is the fraction of first class passengers who survived, that is
$P(\text{passenger survived}|\text{passenger was in first class}) = \frac{201}{324} \approx 0.620.$
- The probability that the passenger survived, given that the this passenger was in second class, is the fraction of second class passengers who survived, that is
$P(\text{passenger survived}|\text{passenger was in second class}) = \frac{118}{284} \approx 0.415.$
- The probability that the passenger survived, given that the this passenger was in third class, is the fraction of second class passengers who survived, that is
$P(\text{passenger survived}|\text{passenger was in third class}) = \frac{181}{709} \approx 0.255.$

These probabilities suggest that the chance of surviving depended on the class, with the first class passengers having the greatest, and the third class passengers having the smallest chance of surviving. Now we want to investigate if what appears to point to class discrimination could be explained in terms of gender of passengers.

First, we ignore the class and take into consideration only the gender of the passengers. We can calculate the following conditional probabilities to compare the probabilities of surviving for a randomly selected child, woman, and man.

- The probability that the passenger survived, given that the this passenger was a child, is the fraction of children who survived, that is:
$P(passenger survived | passenger was a child) = \frac{56}{107} \approx 0.523.$
- The probability that the passenger survived, given that the she was a woman, is the fraction of women who survived, that is:
$P(passenger survived | passenger was a woman) = \frac{313}{417} \approx 0.751.$
- The probability that the passenger survived, given that the he was a man, is the fraction of men who survived, that is:
$P(\text{passenger survived} | \text{passenger was a man}) = \frac{131}{793} \approx 0.165.$

These probabilities suggest that gender was an important factor with rescue procedures, with both women and children having a larger chance of surviving than men.

Now we look at gender distribution between the three classes. Since women and children had large chance of surviving, we can consider them together and calculate the following conditional probabilities:

- The probability that the passenger was a child or a woman, given that the this passenger was in first class, is the fraction of first class passenger who were children or women, that is:
$P(\text{passenger was child or woman} | \text{passenger was in first class}) = \frac{148}{324} \approx 0.457.$
- The probability that the passenger was a child or a woman, given that the this passenger was in second class, is the fraction of second class passenger who were children or women, that is:
$P(\text{passenger was child or woman} | \text{passenger was in second class}) = \frac{117}{284} \approx 0.412.$
- The probability that the passenger was a child or a woman, given that the this passenger was in third class, is the fraction of third class passenger who were children or women, that is:
$P(\text{passenger was child or woman} | \text{passenger was in third class}) = \frac{259}{709} \approx 0.365.$

Looking at these probabilities we can see that there were larger proportions of children

and women in first and second class, than in third class. Now the question is if the difference in gender distribution together with different survival rates for different genders was the only reason to explain the different survival rates for different classes. If that were the case, that is, if class was not a factor in rescue procedures, then any child, regardless of the class in which the child traveled, would have roughly the same chance of surviving (≈ 0.523). The same should hold for all women and all men. Thus we compare the survival rates for passengers of the same gender, but from different classes. First, consider children:

- The probability that a child survived, given that the child was in first class:
$P(\text{child survived}|\text{child was in first class}) = \frac{4}{5} \approx 0.800$.
- The probability that a child survived, given that the child was in second class:
$P(\text{child survived}|\text{child was in second class}) = \frac{22}{22} \approx 1.0$.
- The probability that a child survived, given the child was in third class:
$P(\text{child survived}|\text{child was in third class}) = \frac{30}{80} \approx 0.375$.

We can see that the children in first and second class had a larger chance of surviving than the children in the third class.

We can do similar calculations for women and men.

- The probability that a woman survived, given that she was in first class:
$P(\text{woman survived}|\text{woman was in first class}) = \frac{139}{143} \approx 0.972$.
- The probability that a woman survived, given that she was in second class:
$P(\text{woman survived}|\text{woman was in second class}) = \frac{83}{95} \approx 0.874$.
- The probability that a woman survived, given she was in third class: $P(\text{woman survived} | \text{woman was in third class}) = \frac{91}{179} \approx 0.508$.
- The probability that a man survived, given that he was in first class:
$P(\text{man survived}|\text{man was in first class}) = \frac{58}{176} \approx 0.330$.
- The probability that a man survived, given that he was in second class:
$P(\text{man survived}|\text{man was in second class}) = \frac{13}{167} \approx 0.078$.
- The probability that a man survived, given he was in third class:
$P(\text{man survived}|\text{man was in third class}) = \frac{60}{450} \approx 0.133$.

The final conclusion: The survival rates for women (0.751) and children (0.523) were larger than for men (0.1651), which suggests that the rescue procedures favored women and children. However, a random passenger in first class of any gender had at least twice as large of a chance of surviving as a passenger of the same gender in third class. For example, 0.972 survival rate for women in first class compared to 0.508 survival rate for women in third class. Such discrepancy cannot be justified with different gender distribution between the three classes. Therefore, the given data also

suggests that the rescue procedures favored the first class passengers.